A Study Guide for Electromagnetics

(and antennas)—with SuperNEC *applications*

Alan Robert Clark

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33 Thora Crescent
Wynberg
Johannesburg
South Africa.
www.poynting.co.za

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Dedication

TO MY WIFE, LESLEY JOY, my son, Robert James (4), my daughter, Kathleen Brenda (2): who put up with an irate, busy, me, for the many months it has taken to write this book (Yes, I'll definitely be finished by x, well, y then, Oh!, but by z surely!, Um, z++?).

To Braam Moolenaar for the most powerful editor any programmer could wish for: `vim()` available from `www.vim.org`.

To Thomas Esser for collecting most of what I needed into the decent teTEX distribution, available from `www.tug.org/tetex`.

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To Brian W. Kernighan for the most excellent "Graphics Language for Typesetting" (PIC). Yes, all my graphics is expressed in a programming language. He is also widely rumoured to have invented an unknown, highly obscure language, known simply as "C".

To Dwight Aplevich for overlaying PIC with his brilliant set of "Circuit Macros", available from `www.ece.uwaterloo.ca/~aplevich/Circuit_macros`

To Donald Knuth for the understanding that typesetting is *programming*.

To my colleague, Professor André Fourie, for encouragement and believing in the project. However, if anyone ever says to me again that: "You've got all the background, it'll take less than three months", I'll buy an unused cold-war Nuclear Weapon and throw it at them. It'll be cheap at the price :-)

To 12 years' worth of students to whom I have inflicted an Electromagnetic education. (Maybe).

But most of all to my wife for understanding the engineering (aka JIT) approach to life—I love you.

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Preface

This book has arisen from the need for *visualisation* of difficult concepts in electromagnetics, and in antenna design. A grasp of the *fundamentals* of electromagnetic theory is *essential* before application can begin in earnest. Often these fundamentals are stated simply as equations—and with some of the equations in electromagnetics being rather hairy, the student is lost.

Over the many years of teaching electromagnetics at an undergraduate level, I have tried many forms of visualisation to get complex concepts across, but none is so effective as the use of SuperNEC.

It is my hope that this book will be an aide to the student of electromagnetics: it is purposefully not pitched at any particular textbook of electromagnetics and it not meant to be one. It is, in short, a study guide.

About the Author

Alan Robert Clark was born in Derby, England in 1964. He "emigrated" to South Africa in 1969, where he lives with his wife and two children. He obtained a B.Sc(Eng) Elec. in 1987, and his Ph.D in 1993, all from the University of the Witwatersrand, Johannesburg. He is a registered Professional Engineer.

He is an Associate Professor, lecturing Electromagnetics at the School of Electrical and Information Engineering `www.eie.wits.ac.za`, at that university. He also teaches Electronics.

He has consulted widely in industry, designed many antennas and associated electronic devices, has published several papers, and can be found at `ytdp.ee.wits.ac.za`

Chapter 1

Transmission Lines

This chapter briefly overviews transmission line theory, demonstrating that the voltage and current down the transmission line changes as a function of distance down the transmission line, resulting in a change of *impedance*. It then looks at impedance matching techniques, to minimise the reflection of power from its intended destination: the load.

1.1 Transmission line theory

TRANSMISSION lines connect *generators* to *loads* as shown in fig 1.1. In the RF world, in the transmitting case, this is viewed as connecting the transmitter to the antenna, and in the receiving case as connecting the antenna to the receiver.

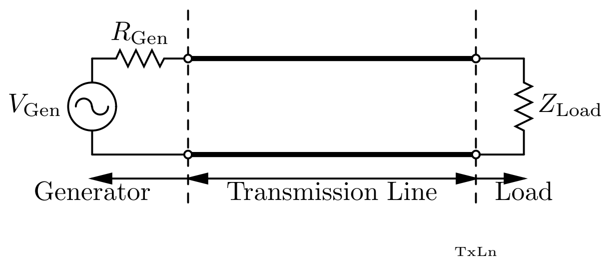

Figure 1.1: A transmission line connects a generator to a load.

From a standard circuits analysis perspective, the transmission line simply consists of connecting two parts of the circuit, and does not change anything. By Kirchhoff's Voltage law, there is no change in voltage or current along the length of the "connection".

As a rule of thumb, as soon as the "connection" length between the parts of the circuit exceeds a fiftieth of a wavelength ($\lambda/50$), transmission line theory must be applied, and circuit theory breaks down.

It is the *length* of the "connection" that causes a finite *time* delay to occur in getting from one end to another—which is the same as saying that there is a *phase* difference which has occurred from one end to the other. Thus, length, time, and phase are synonymous within a transmission line, all as a function of the *wavelength* of operation of the line.

Recall that the *free-space* wavelength, λ, is simply given by the useful approximation:

$$\boxed{\lambda(\mathrm{m}) = \frac{300}{\mathrm{f(MHz)}}} \tag{1.1}$$

1.1.1 Transmission lines as Lumped Circuit Elements.

In circuit terms, the distributed capacitance and inductance etc of the line can be collected in lumped models. The model of the transmission line is then an infinite set of these circuit sections. Many versions of the model exist, but I shall use the standard Kraus and Fleisch [1999] model, as shown in fig 1.2.

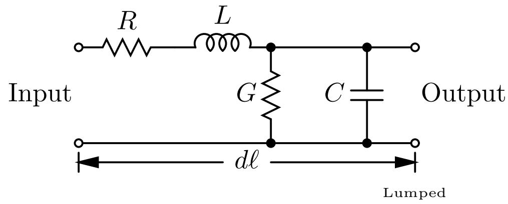

Figure 1.2: A "Lumpy" model of the TxLn, discretizing the distributed parameters.

For a given length of transmission line, we can hence lump the series resistance R [Ω/m] and inductance, L [H/m] together; and the shunt conductance G [℧/m] and capacitance C [F/m]. These terms are per-unit length, and do not change from one section of the transmission line to another (uniform transmission line). Hence we define a *characteristic impedance*, Z_0, as the ratio of the series to the shunt components; in the lossless (or high frequency) case, R and G are negligible:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad \left(= \sqrt{\frac{L}{C}} \quad \text{Lossless} \right) \tag{1.2}$$

The velocity with which the wave moves down the (lossless) transmission line is also dependant on the material properties of the medium:

$$v = \frac{1}{\sqrt{LC}} \quad \text{m/s} \tag{1.3}$$

The velocity of actual propagation down the line as a fraction of the speed of light is what we are usually interested in, called the Velocity Factor (VF):

$$\mathrm{VF} = v/c \tag{1.4}$$

As v *will always* be less than c (speed of light in free-space), the *physical length* of a transmission line is always *less* than its *electrical length*.

$$\boxed{\ell_{\text{phys}} = \text{VF} \times \ell_{\text{elec}}} \tag{1.5}$$

The Z_0 for a particular type of transmission line is thus derived by getting expressions (or measurements) of the per-unit length capacitance and inductance etc. For the simplest case of a two-wire line, this is [Wadell, 1991, pg66]:

$$Z_0 = \sqrt{\frac{\mu_0 \mu_r}{\pi^2 \epsilon_0 \epsilon_{r(\text{eff})}}} \cosh^{-1}\left(\frac{D}{d}\right) \tag{1.6}$$

Since magnetic materials are never used, the simplified equation is usually derived:

$$\boxed{Z_0 = \frac{120}{\sqrt{\epsilon_{r(\text{eff})}}} \ln\left(\frac{D}{a}\right)} \tag{1.7}$$

where $\epsilon_{r(\text{eff})}$ is the *effective* permittivity (usually some air and some plastic).

1.1.2 Impedance Transformation

Thus, for a loaded transmission line, the input impedance Z_{in} is a function of how mis-matched Z_L is from the ideal of Z_0, and a function of the (electrical) length of the line.

$$\boxed{Z_{\text{in}} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell}\right]} \tag{1.8}$$

This equation is best solved using the Smith Chart, which is a plot of the Voltage Reflection coefficient ρ (sometimes called Γ in some texts) in the complex plane, with constant resistance and impedance circles superimposed on it.

The extent to which power is reflected from the load is dependant on how “bad” the load mismatch is to the line characteristic impedance:

$$\boxed{\rho = \frac{Z_L - Z_0}{Z_L + Z_0}} \tag{1.9}$$

The mismatch in impedance is also often stated in terms of the voltage standing wave ratio (VSWR) on the transmission line.

$$\boxed{\text{VSWR} = \frac{1 + |\rho|}{1 - |\rho|} \quad \left(= \frac{V_{\max}}{V_{\min}}\right)} \tag{1.10}$$

Useful Special cases of the transmission line equation:

1. $Z_L = Z_0$ This condition results in: $Z_{\text{in}} = Z_L = Z_0$, regardless of line length, or frequency. This is the *matched* case.
2. $\ell = \frac{\lambda}{2}$. $Z_{\text{in}} = Z_L$ regardless of characteristic impedance. This is the *halfwave* case.
3. $\ell = \frac{\lambda}{4}$ "Quarter wave transformer" case. $\boxed{Z_{\text{in}} = \dfrac{Z_0^2}{Z_L}}$

   This configuration is useful since it can transform one load impedance to a different one if a line with the correct impedance can be found.
4. Open or short circuited lines.

   $Z_{\text{in}}(\text{oc}) = -jZ_0 \cot \beta\ell$ for an open circuited line

   $Z_{\text{in}}(\text{sc}) = jZ_0 \tan \beta\ell$ for a short circuited line

   In both these cases the impedance is purely reactive and if the lines in question are less than a quarter wave it is clear that such lines could be used to "manufacture" capacitive (open circuit case) or inductive (short circuit case) reactances. It should be remembered however that the capacitance or inductance of such a line would itself be frequency dependent.

The open and short circuit cases provide a convenient way to *measure* the characteristic impedance of a line, since combing them yields:

$$\boxed{Z_0 = \sqrt{Z_{\text{in}}(\text{oc})Z_{\text{in}}(\text{sc})}} \tag{1.11}$$

The velocity factor of a line can be measured by using the quarter-wave transformer principle—if the load end is open circuited, $Z_L = \infty$, hence Z_{in} =0! The method is then to take an open-circuited line and measure the input impedance, increasing the frequency until the input impedance drops to a minimum. The line is then at an electrical quarter-wavelength, so

$$\text{VF} = \frac{\ell_{\text{phys}}}{\lambda/4}. \tag{1.12}$$

1.2 Exercises

Exercise 1.1: The `sgtxln` assembly: changing the physical length

Purpose: *Gentle introduction to SuperNEC, and the Smith Chart, including overlaid plots, as well as the concept of impedance change as the length of the transmission line changes (physically).*

There are two different ways of using a transmission line in SuperNEC. The usual method is to highlight the two segments on your antenna, and to use a "TL card"—a mathematical transformation—obtained under `Add| Primitive| Network| Transmission Line` on SuperNEC's menu.

But it is also possible to model a two-wire line physically using wire segments in SuperNEC, and this is done in the `sgtxln` assembly. There are limitations to this method as SuperNEC's modelling guidelines are violated :

- The line cannot be too long, 2 wavelengths seems to be the limit.
- The lines cannot be too close, putting a lower limit to the characteristic impedance, Z_0, at 200Ω.
- The lines cannot be too far apart (since they begin to radiate) limiting the upper end of the characteristic impedance to about 600Ω.

1. Pull up the `sgtxln` , shown in figure 1.3

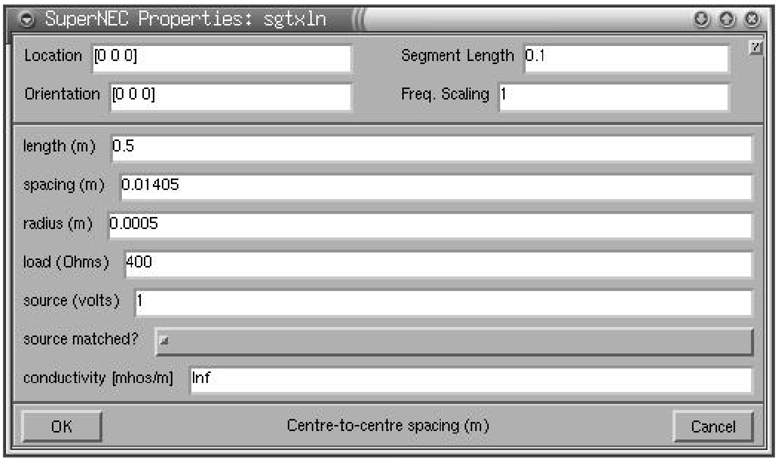

Figure 1.3: Dialogue box for the `sgtxln` transmission line assembly

   Change the length default to 0.01 (very short) and change the load default to 800Ω, hit the simulate button.
2. The output viewer will pop up as shown in fig 1.4

   Click on the Parameter vs Frequency tab, and choose the excitations for model 1, and click plot.

   You will notice that there is only one point on the Smith Chart, far away from the 2:1 VSWR circle. The default Zo for the Smith Chart is 50Ω, whereas our line is a 400Ω line. From the menus, Choose `Options|Zo...` and fill in 400.
3. *Do not close the output viewer or the Smith Chart Plotter.*

   Go back to the input interface, click on Select All, and Edit, changing only the length to 0.1m. Re-simulate, and from the output viewer, choose *Model 2*, Parameters vs Frequency and the Excitations for Model 2. Then click the Overlay button before clicking on Plot. A second point is now plotted on the Smith Chart.

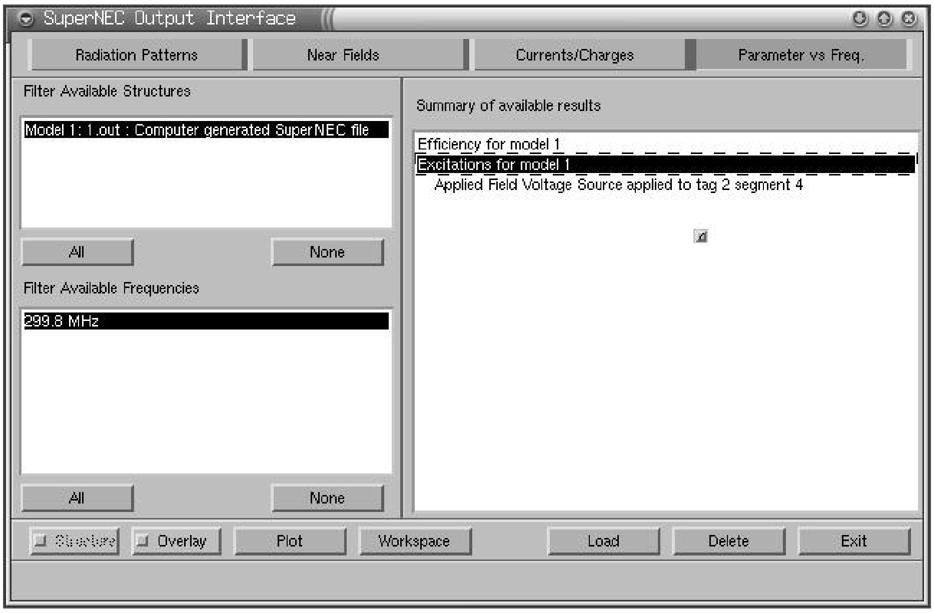

Figure 1.4: Output Viewer interface

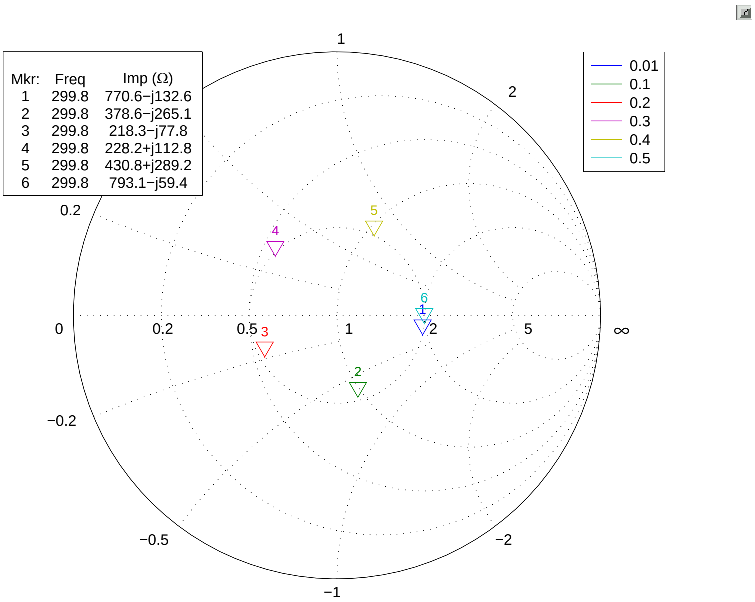

Figure 1.5: Smith Chart plot showing the impedance change for several different lengths of transmission line

4. Repeat the above for 0.2, 0.3, 0.4 and 0.5m transmission lines. You should have a plot that looks like fig 1.5

   Note that the points at 0.01 and 0.5 m are nearly coincide. *Every feature* *of the transmission repeats itself every half wavelength* For this reason, a full revolution around the Smith Chart represents a half wavelength move down the line.

Note too that the impedances obtained while moving down the line are not simply arbitrary—one cannot obtain an impedance of $400 + j800$ on this particular line. The available impedances are constrained by the VSWR circle that one is traversing.

Conclusion: As the line has become physically longer, the impedance seen by the source changes. When the line is very short, it is close to 800Ω, but as soon as the line becomes appreciably long, the impedance becomes capacitive. Note however, that all the points lie on the 2:1 VSWR circle centred on the chart.

Exercise 1.2: The `sgtxln` assembly: changing electrical length

Purpose: *To demonstrate that a frequency sweep on a fixed length line varies its length electrically, and this is seen as a "walk down" a constant VSWR circle. To introduce aspects of the impedance plotters.*

1. Pull up the `sgtxln` assembly, and accept the change the load Z_l to 800Ω to produce a 2:1 VSWR. The other defaults can be left, which produces a 0.5m long 400Ω line. Note that at the default model frequency of 300MHz, 0.5m is exactly 0.5λ long electrically.
2. From the input editor, use the `Edit|Simulation Settings` dialogue box, as shown in fig 1.6

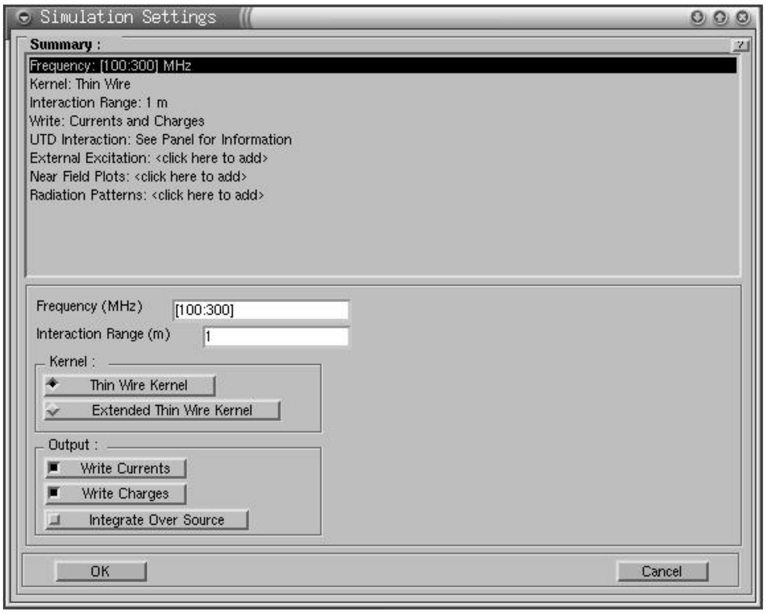

Figure 1.6: Simulation settings dialogue

Change the frequency sweep to `[100:300]`. *NOTE: your highest frequency* *specified in the simulation MUST ALWAYS be equal or less that your MODEL FREQUENCY shown on the input editor*

3. After pressing the `Simulate` button, clicking on the `Parameter vs Freq` tab in the output viewer, selecting the excitations from the appropriate model, plotting, and finally setting the default Z_0 as shown in the previous exercise, you should have a Smith Chart as shown in fig 1.7

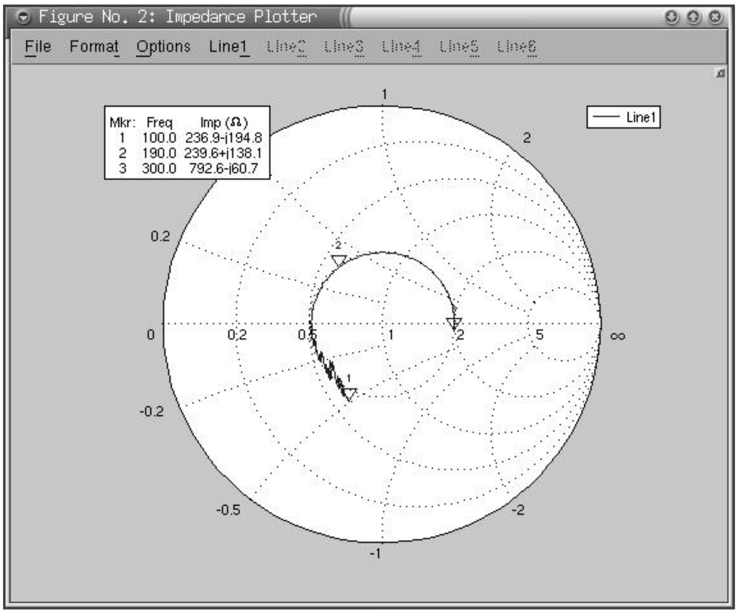

Figure 1.7: Smith Chart of a frequency swept transmission line

By clicking on the plot, markers are placed on the plot, and the Marker legend box is automatically drawn. Fig 1.7 shows three such markers, complete with the impedance at that point and the frequency.

Markers are extremely useful on Smith Charts, as there is no natural indication of frequency on the chart itself, in some plots, the frequency intervals are close, resulting in a dense collection of points, and sometimes far away. Markers allow that judgement to be made.

Notice that all impedance points stick to the 2:1 VSWR circle, with a little instability at the low frequency end, where some modelling guidelines are violated.

4. Choose `Format|VSWR` to show the plot as a function of frequency, plotting VSWR. Note that all markers are retained etc, as shown in fig 1.8
5. Choose all the other possible formats of representing the impedance output.

Conclusion: The Smith Chart gives a very good overview of the entire impedance spectrum, which is why it is the most useful means of representing a frequency swept transmission line. This becomes important when "real" loads are used such as dipoles etc. However, other forms of representing the impedance representation can often give different insights.

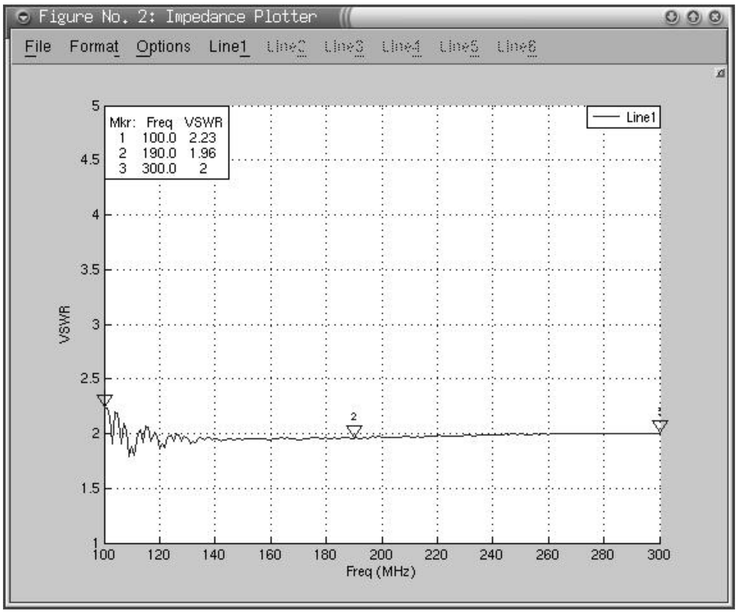

Figure 1.8: VSWR plot vs frequency of the frequency swept line.

Exercise 1.3: The sgtxln assembly, introducing lossy lines

Purpose: *To demonstrate that a frequency sweep on a lossy line no longer sticks to a constant VSWR line. To introduce various lower level editing facilities of* SuperNEC.

There are a number of ways of introducing loss to a transmission line (Wire conductivity or physically loading the wires), but none address the G shunt conductivity element in the lumped model shown in fig 1.2. Changing the conductivity or adding resistance is a good approximation though, and this exercise will use actual loads as a means of introducing level-specific editing facilities in SuperNEC.

1. Pull up the `sgtxln` assembly as usual, changing the load Z_l to 800Ω. You will notice that the Group Level in the left bottom corner is set to "high". Pushing the `Edit` button at this points brings up the `sgtxln` dialogue, ie the highest level assembly.

   But `sgtxln` itself is made of `snwire` objects, and each `snwire` object is in turn made up of segments, which are the lowest level primitives in *SuperNEC*. Click on the "<" button changes the "high" to "2" denoting an intermediate group level.

   After Unselecting All, clicking on one of the longer wires selects all segments in that wire. The `Edit` button now pulls up the `snwire` dialogue box.

Click on the "<" and the Group Level now shows "low" meaning that the lowest, or primitive level has been reached. Clicking on one of the longer wires now selects only one segment, and the `Edit` button pulls up the segment dialogue box.

NOTE: Any changes made at the lower levels are not preserved at the higher level. If a segment is deleted at the "low" level, any re-segmentation at the assembly level "high" will re-create it.

2. Get back to Group Level "2", and select both longer wires of the `sgtxln` assembly. From the Menu, choose `Add| Primitive| Load` and change the resistance to 10Ω.
3. Edit the Simulation Settings to simulate from 100 to 300MHz, and simulate, and you should get the Smith Chart shown in fig 1.9

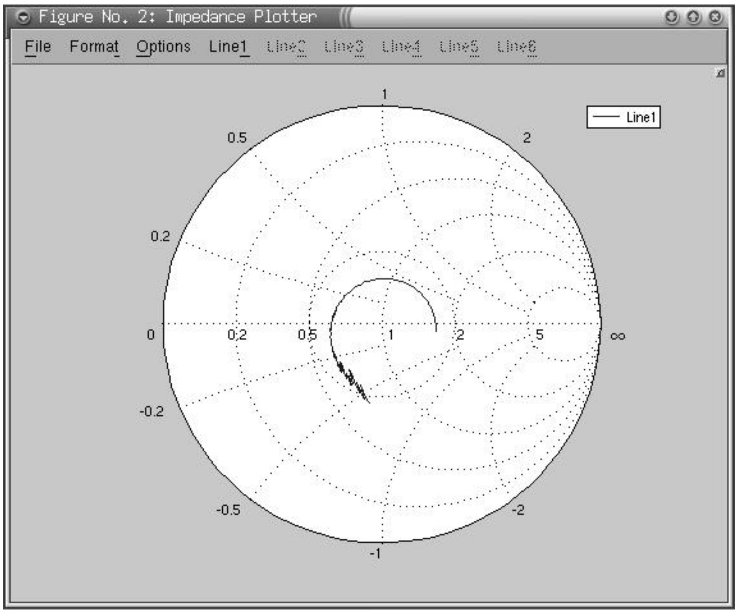

Figure 1.9: Lossy line frequency sweep.

The classic "spiralling in" as the VSWR seen by the source gets progressively better as the amount of loss increases due to the effective lengthening of the line. A VSWR plot shows the effect quite nicely.

Of course, the SuperNEC model does not capture the shunt loss, and the assumption that the $Z0$ is still 400Ω in the Impedance Plotter is incorrect, showing some of the limitations of the simulation. Entering a Z_0 of $400 - j15$ improves matters. (Hand calculation of the Z_0 at 100MHz yields $401 - j23.8$ and at 300MHz, $400.3 - j7.95$ Note that in a lossy line, Z_0 changes with frequency!)

Conclusion: Lossy lines can be modelled, within certain constraints, with the `sgtxln` assembly within SuperNEC and reasonable results obtained.

Exercise 1.4: Current Magnitudes down a transmission line

Purpose: *To illustrate the standing wave in the current magnitudes on a transmission line as a function of the level of mismatch on the line.*

1. Pull up the `sntxln` dialogue box and generate a 1 wavelength long transmission line (1m at the default 300MHz), terminated in 400Ω (ie matched), with the `source matched?` check-box checked. Set the model frequency to 1500MHz, even though the simulation will only run at 300MHz, so that a decent current resolution is obtained.

   NOTE: It is important for this entire exercise to make sure that the source is matched to the transmission line. Normally we do not bother with this, but the absolute current magnitudes calculated for this exercise differ widely if the source isn't matched, making meaningful comparisons difficult. (The shape is still the same, the magnitudes just don't make sense between the various runs)
2. Simulate, and select the Current Distribution(s) for model 1 in the output viewer. If you plot this, you will see a current variation along the line, but this is deceptive as you will see that the bottom of the scale is 1.1 and the top 1.2 ($\times 10^{-5}$).
3. Store the current distribution in a workspace variable for later processing: click the `Workspace` button in the Output Viewer, and change the default variable name `towork` to `a`.
4. If you inspect what you now have in the workspace, you will find that the variable `a` is a struct array containing the top level elements `a.currents` and `a.structure`. We are only concerned here with the `a.currents` branch of the struct.

   `a.currents` is further broken down into `a.currents.freq` and `a.currents. currents`. We are ultimately interested in `a.currents.currents`—but only the first half of them! (the second half is a repeat of the currents on the "other" wire, which we are not interested in. We will deal with these later in this exercise.
5. Go back to the input interface and click on `Select All`, and `Edit`. Change the load to 800Ω. Re-simulate and store the currents in workspace variable `b`.
6. Repeat for 200Ω, storing the currents in `c`.
7. Repeat for a short circuit (0Ω), store in `d`.
8. Repeat for an open circuit:"Inf" does not work, as the result is a collection of NaN's. The Simplest way is to select the lowest level of editing in the input editor, using the < button. Select the terminating load and click the `Delete` button. Store the results in `e`.
9. Plot the current distributions using the purpose-built m-file sgtxlnplotcurrents, passing all the saved "to Workspace" structs as a vector, ie:

   ```
   sgtxlnplotcurrents([a,b,c,d,e]);
   ```

   The result is shown in fig 1.10, which shows that under matched conditions (a), the current down the line is roughly constant—there is no mismatch a the load, no reflected wave, and hence no standing wave on the line. This ideal case only has a travelling wave.

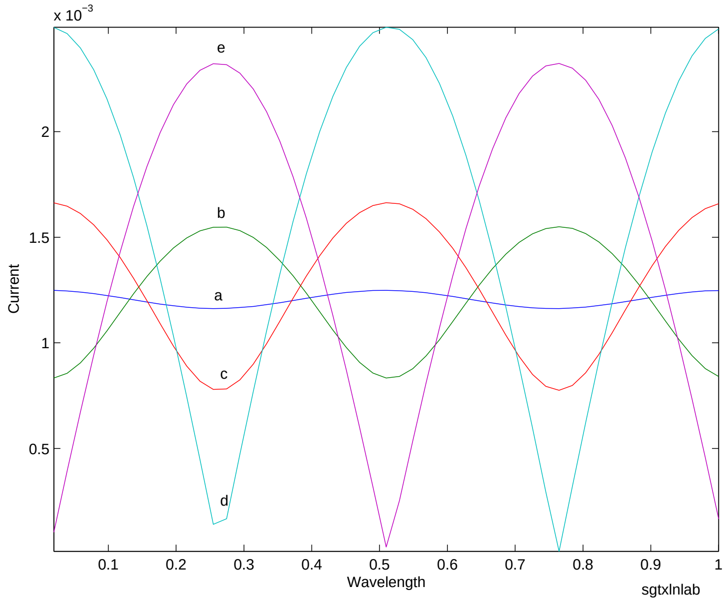

Figure 1.10: Current magnitudes on a transmission line with different mismatches

In case (b), a 2:1 mismatch was applied with the load being twice the characteristic impedance of the line. It can be seen that there is a standing wave on the transmission line, due to the reflection at the load.

Note that the deviation of (b) above (a) equals the amount of deviation of (b) below (a). ie the amount of constructive interference is equal to the amount of destructive interference.

Note that the points of constructive and destructive interference are a quarter of a wavelength apart.

Finally note that the points of constructive interference are a half a wavelength apart. This is an accurate way of measuring the wavelength in a microwave waveguide, for example, where a little probe is slid along the length of the waveguide, plotting a similar pattern.

In case (c), a similar 2:1 mismatch is applied, but this time with *half* of the characteristic impedance. Note that the points of constructive and destructive interference are exactly opposite to those of (b), but that the magnitude of the deviation from (a) is roughly equal to that of (b): the smaller the mismatch, the closer to (a) you get!

Finally, then, in cases (d) and (e), we have the ultimate mismatches—short and open circuit respectively, showing very nice current nulls. Note that the conditions of open and short circuit interchange every quarter wavelength (min and max current, respectively), which demonstrates very nicely the conversion from an open to a short circuit for a quarter wavelength line. Note too, that a "rectified" sine shape is obtained—it is *magnitude* after all!

Conclusion: This exercise demonstrates very visually the effect of mismatching a transmission line on the current magnitudes on the line. It demonstrates that everything on a transmission line repeats itself every half-wavelength, that maxima and minima are quarter wavelengths apart, that the degree of mismatch influences the degree of standing wave, away from the ideal "flat line", and finally that an open circuit does indeed convert to a short circuit a quarter-wavelength away.

The inspiration for this particular exercise comes from the "transmission-line lab" that I have run for years on end at Wits: there we have a lumped model of a transmission line (in 17 lumped sections), and the Voltage standing wave is measured by oscilloscope at every node. SuperNEC is more current-centric, but it illustrates the same thing in a very visual manner. One thing the physical lab does though is that you still have a time-oscillating sine-wave on the oscilloscope at each node: what fig 1.10 shows is the spatial amplitude of the envelope of the standing wave. Don't forget that at all points shown, there is still a time-oscillation!

Exercise 1.5: Determination of characteristic impedance.

Purpose: *To illustrate that the characteristic impedance is the square of the short-and-open circuit input impedances of a transmission line.*

1. Pull up the `sgtxln`dialogue box, modifying the length to 1.12m (yes, a well-cooked number), ensure that the source is *not* matched to the transmission line, and enter the load as 0 Ω (a short-circuit)
2. Simulate and plot the single impedance point on the Smith Chart. Leave the Smith Chart Viewer window open.
3. Go back to the input editor and use the < button to set the `Group Level` to "low". Select the load segment only (the black blob is the load, the orange blob is the source end—don't kill that!) and click the `Delete` button. The transmission line is now effectively open circuited.
4. Simulate and overlay the plot of the input impedance, obtaining something like fig 1.11.

   Taking these figures into the workspace, we get:

```
>> sqrt((0.5+426.4i)*(0.4-401.7i))

ans =

  4.1387e+02 - 3.6594e-02i

>>
```

   ie $414 - j0.04\Omega$, a fair approximation to 400Ω!
5. Using the menu `Options| Zo` dialogue to enter 414Ω instead of the default 50Ω, you will see the markers close to $0 + j1$ and $0 - j1$, which will give

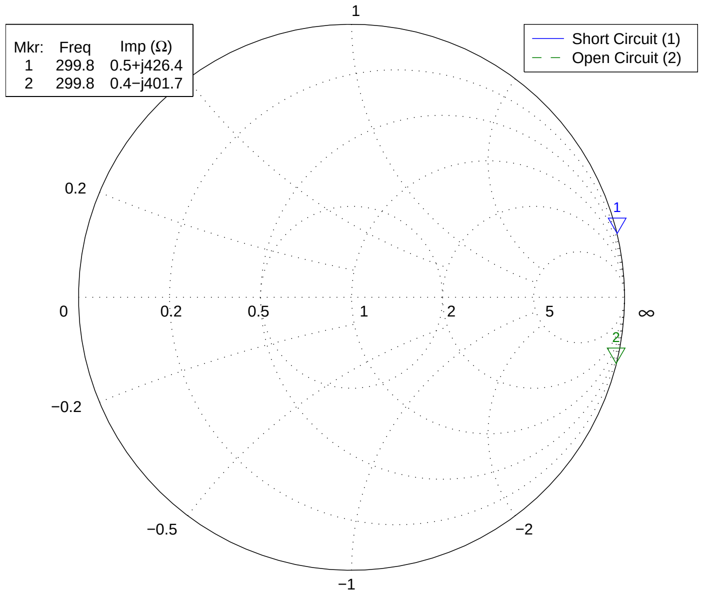

Figure 1.11: Open and Short circuit input impedances for characteristic impedance determination.

lower error than anywhere else on the chart—hence the carefully cooked length in the beginning of the exercise!

Conclusion: It is possible to show that the characteristic impedance of a transmission line is the geometric mean of the transmission lines' open and short circuit input impedance, using SuperNEC.

Exercise 1.6: Determination of Velocity Factor (VF).

Purpose: *To illustrate that a transmission line whose conductors are covered in a dielectric sheath has a slower velocity of propagation than an open-wire line, and to determine what that difference is, as a factor.*

1. Using the `sgtxln` assembly as usual, make it 1m long, check the checkbox to ensure that the source is matched to the transmission line, set the `Model Frequency` to 1500MHz for decent current-plot resolution, click < until the `Group Level` is "low", select just the load (black blob) and click `Delete`. As before this produces an open circuited transmission line.
2. Simulate (at the default of 300MHz), and store the current distribution as `a` by using the `Workspace` button in the Output Viewer.
3. Go back to the Input Editor and raise the `Group Level` setting to "2", halfway between "low" and "high". Select one of the long transmission line wires. Pressing the Shift key, select the other long wire.

   Using the menu `Add| Primitive| Load` (which will add whatever you specify to *all* selected segments), choose a `Wire Sheath`; specifying 0 conductivity, 2.3 for the relative permittivity, and 0.005m for the thickness. (The thickness just exaggerates the effect, one would not normally have such a thick layer of plastic (2.3))

4. Simulate, store as `b`.
5. From the workspace, run `sgtxlnplotcurrents([a,b])` to get something that should look like fig 1.12

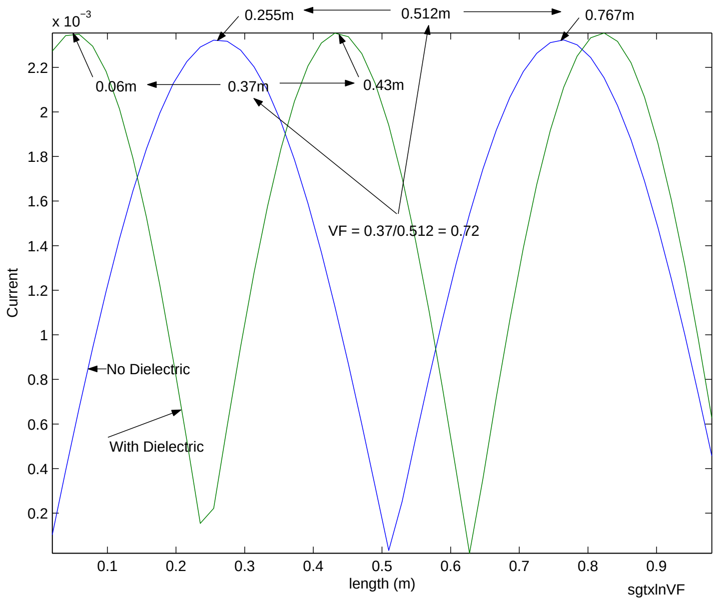

Figure 1.12: The effect of a dielectric sheath on the velocity factor of a transmission line.

Note that although the physical length of the line has not changed between simulations, the electrical length has changed due to the addition of the dielectric. Remember that the fields are bounded by metal structures, but travel in the medium between. Hence a change of medium produces a change in characteristics, including the velocity of propagation down the line.

In the rather thick-skinned plastic-coated transmission line in this example, the speed is reduced to 72% of the open-wire speed.

Conclusion: Within certain modelling limitations, SuperNEC can predict the Velocity Factor (VF) of a two-wire transmission line with a dielectric coating. It is shown that the addition of such a coating slows down the wave, making the wavelength within the transmission line shorter.

1.3 Problems

1-1. Characteristic impedance of a two-wire line Using the equations for a two-wire line, calculate the dimensions for a 300Ω transmission line and implement it. Run through a few of the exercises with this line as e base.

1-2. Characteristic impedance of a two-wire line Investigate just how far you can push the modelling guidelines in terms of achieving various characteristic impedances of two-wire lines. Plot your best attempt at a matched 50Ω line!

1-3. Dielectric Sheath modification of transmission line characteristic impedance Repeat the exercise 1.2, but use the method shown in exercise 1.2 to determine the characteristic impedance of the dielectric clad transmission line.

Attempt to relate the change in characteristic impedance to the ϵ_{eff} term in the two-wire characteristic impedance equation—ie what would you calculate ϵ_{eff} to be in this case?

1-4. Dielectric Sheath on 300Ω "Tape" Introducing a dielectric sheath has two effects: it changes the velocity factor *and* the characteristic impedance. Another difficulty is that the thickness of sheath may have to be artificially increased to account for the plastic "web" holding 300Ω tape together.

Thickness, relative permittivity both affect both the Z_0 and the VF. By Iteration, attempt to find values that will deliver a model for 300Ω "tape"

1-5. Creating Assemblies Create a new assembly, `sgtxlndiel`, using `sgtxln` as a starting point, to more easily allow, from the main dialogue box, a dielectric coating to be specified for the transmission line.

1-6. Creating Assemblies Create a new assembly, based on `sgtxln`, that allows the creation of a cascaded transmission line: The first section being of different characteristic impedance to the second section. Note that it is the radius of the wires that will have to be changed, not the spacing between them. What are the limitations of this arrangement?

Chapter 2

Matching

This chapter introduces the concept of matching, the rationale behind wanting the system to be matched, and various simple matching techniques. These techniques are then applied to various real-world examples. It also introduces the lower-level features of SuperNEC which enable some powerful visualisations.

2.1 Theory

2.1.1 Standing Waves

The interaction between the forward travelling wave and the reflected travelling wave results in a standing wave on the transmission line. The standing wave consists of the constructive and destructive interference of the two travelling waves. The amount of interference is directly proportional to the amount of reflected power, which in turn, is directly related to how badly matched the load is to the transmission line. The reflected power obviously results in a loss of power actually delivered to the load (antenna).

Impedance transformation has the goal of *matching* the antenna impedance to the characteristic impedance of the transmission line, and to the source impedance of the generator in order that *no reflected power* exists—maximum power transfer is the desired goal.

A transmitter cannot deliver maximum power to an unmatched load, but this is not the only consideration:

- High VSWR means high $V\&I$ at various points on the transmission line, thus increasing the transmission line losses at those points.
- High V may mean flashover or dielectric breakdown in high power systems.
- High I may mean hotspots or copper melting.
- Output electronics of the transmitter can be damaged, or more likely, the automatic power reduction circuitry kicks in. (Typically at a VSWR of 2:1)

Note that ISWR = VSWR, hence a 2:1 VSWR means that at some point on the line there is a voltage twice as high as at another point, and since the power remains constant, the low voltage point will have a current twice as high as the current at the high voltage point. Thus a 2:1 VSWR specification means that the output transistor of the transmitter may be asked to deal with twice the voltage ***and*** twice the current, therefore four times the power rating! (in the extreme case of open and short circuit). For this reason, most transmitters detect a VSWR of more than 2:1 and shut themselves down.

2.1.2 Impedance Matching

Impedance matching is important both for transmission and reception. It is more critical, however, for the transmitting case and the VSWR specifications are usually more severe. To illustrate this point the following equation gives the power reduction as result of a mismatch in terms of VSWR:

$$\boxed{\text{Power lost in transfer} = 10\log\left(1-\left(\frac{\text{VSWR}-1}{\text{VSWR}+1}\right)^2\right) \quad \text{dB}} \tag{2.1}$$

Thus a VSWR of 2 : 1 results in a power reduction of only 0.5 dB. Even a VSWR as high as 5 : 1 only causes a reduction of 2.5 dB. The power reduction due to the mismatch condition itself is thus not all that significant, but some transmitters will start reducing power output to protect the driving stage electronics at such low values as 1.5 : 1 or 2 : 1 (Or simply blow up if no power reduction protection is in place). The power lost (in dB) versus VSWR is illustrated in figure 2.1.

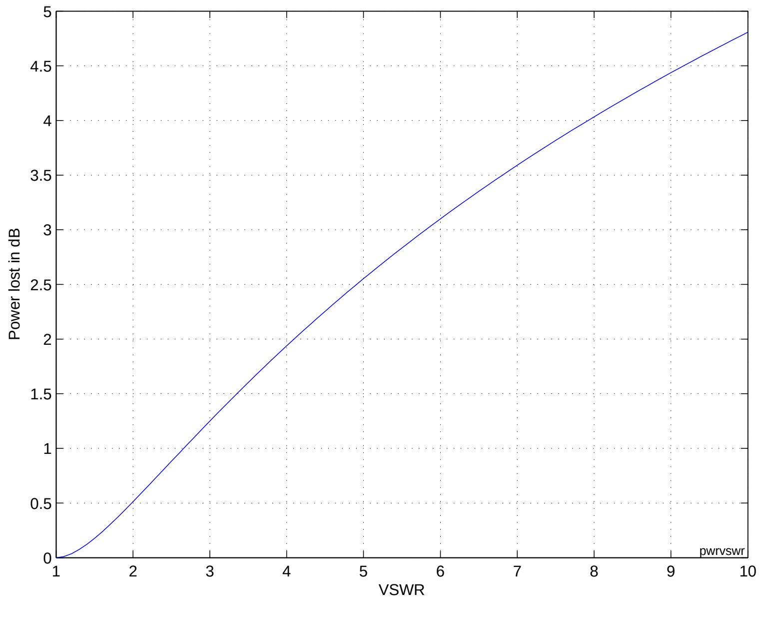

Figure 2.1: Power loss in dB versus VSWR on the line.

(As an aside, remember that maximum power transfer is desirable in RF applications—but I *do not* want maximum power transfer from the local electricity power station to my computer!)

2.1.3 The Quarter wave transformer

Noting that the impedance seen down the length of a transmission line changes, it is often useful to use an appropriate piece of different transmission line (ie different characteristic impedance) to transform the mismatched load. The most common method is the Quarter-wave transformer.

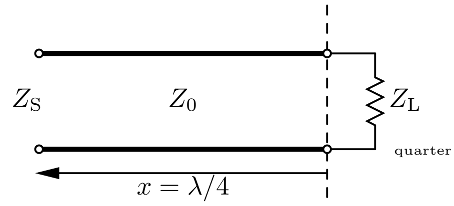

Figure 2.2: Quarter-wave transformer

In this case, the transmission line equation collapses very nicely to:

$$Z_0 = \sqrt{Z_L Z_S} \tag{2.2}$$

which implies that if you can find an appropriate cable Z_0 you can match any system, restricted to the common cable types. In the case of Microstrip lines, however, it is possible to manufacture almost any reasonable Z_0.

Note that because the inserted transmission line section is only a quarter-wavelength long at a particular frequency, the matching technique is narrow-band. However, many communications systems are inherently narrow-band, and the technique is very useful.

2.1.4 Stub match

Another Common technique is the use of short-circuited stubs. Recall that a short-circuited, lossless line provides only reactive components: ie they look like capacitors or inductances. If they are placed at appropriate places along the transmission line, the system can be matched. Almost any system can be matched in this way, but again, since the stubs are of a certain length, this method is also narrow-band.

Since the impedances presented by the stub (Z_{st}) and that of the line (Z_{ln}) are placed in parallel at the junction, we prefer to deal with admittances: (Y_{st}) and (Y_{ln}) since that simply means that the admittance at the junction is:

$$Y_{\mathrm{jn}} = Y_{\mathrm{st}} + Y_{\mathrm{ln}} \tag{2.3}$$

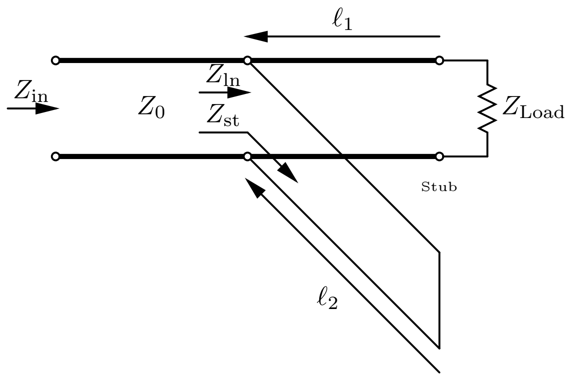

Figure 2.3: Short-circuited Stub match

and it becomes much easier to deal with. Noting that a short-circuited stub *can only* supply reactance, it is *thus required that the real part* of (Y_{ln}) at the point of junction must be what will be *ultimately required*, since the stub cannot change the real part if it can only supply reactance.

The objective is thus to move the load (as an admittance) along a constant VSWR line down the transmission line until the $R = G = 1$ circle is reached on the Smith Chart.

The length of the stub is simply given by the amount of (opposite) reactance that is required at that point on the $R = G = 1$ circle. This will be shown in an exercise.

2.2 Exercises

The principal assembly that we will use is the `sgtl` (Study Guide Transmission Line, or "TL card" in old NEC2 parlance) assembly. Previously the `sgtxln` assembly was used to demonstrate SuperNEC's ability to actually simulate transmission lines as well as radiating structures, but we also showed that modelling guidelines were easily violated if you pushed it too far! Hence, in this chapter, we will use SuperNEC's idealised TL transmission line, which is simply a mathematical transformation, implementing the transmission-line equation (1.8).

Exercise 2.1: Quarter-wave Transformer with simple load.

Purpose: *To illustrate the usefulness of the quarter-wave transformer.*

1. In the simplest case, the quarter-wave transformer simply transforms one value to another. Pull up the `sgtl` assembly and specify a length of 0.25m ($\lambda/4$ at the default frequency of 300MHz), a load of 113Ω (a well-cooked number) and a characteristic impedance Z_0 of 75Ω. The dialogue box that will appear is shown in fig 2.4, and the input viewer will look like fig 2.5.

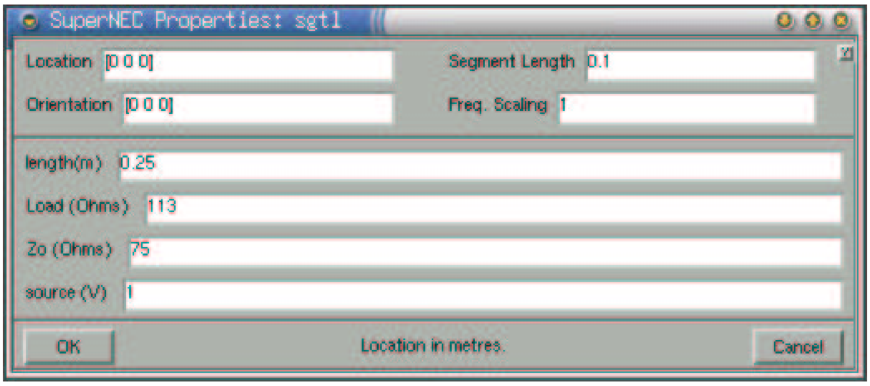

Figure 2.4: Dialogue box of the `sgtl` assembly.

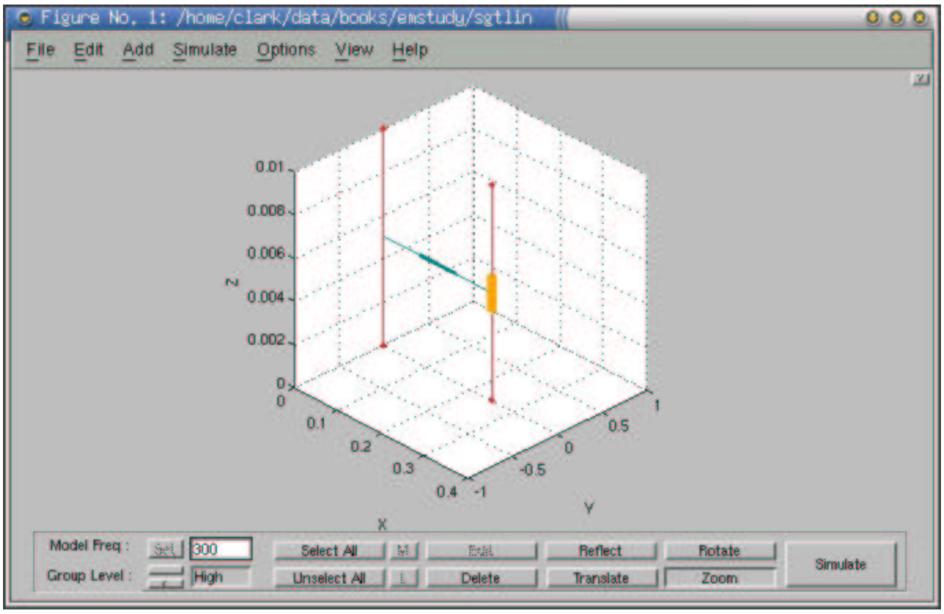

Figure 2.5: Input Viewer with an `sgtl` assembly.

In fig 2.5 you will see two segments since a TL card *must* be connected to a wire segment (even if its only one segment!), a source on the right-most segment, and a transmission line element connecting the two segments. In order to prevent the segments interfering in the SuperNEC simulation if other assemblies were present, you will note that the lengths of the segments is small.

Since a load was specified, you may have expected a load to appear on the left-most segment in fig 2.5, but that causes numerical errors within SuperNEC—the load is incorporated as a terminating condition in the TL card.

2. Run the simulation, and plot the resulting (single-point) impedance, as

shown in fig 2.6.

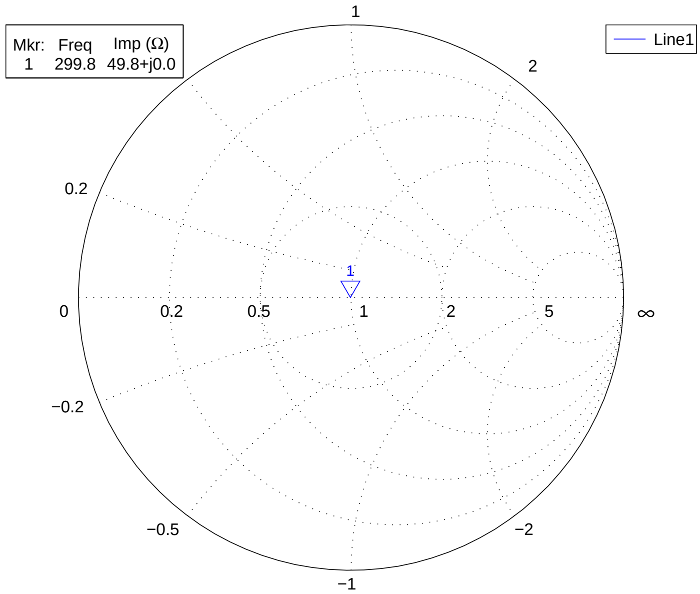

Figure 2.6: Smith Chart plot of single-point output of a simple `sgtl` assembly.

As shown in fig 2.6, the input impedance to this system is pretty much 50Ω.

3. Now add a Frequency Sweep (Still using a pure resistor, which doesn't change with frequency). Obviously the effective electrical length of the transmission line will change as the frequency changes though. Using `Edit| Simulation Settings` change the single frequency to a quite wide sweep of `[50:600]`. Note that, theoretically, one should change the `Model Freq:` entry in the Input Viewer, but since the segments are so short anyway, it does not re-segment the model!
4. Simulate and plot the VSWR (from the `Format` menu item on the Impedance plotter), and check the 2:1 VSWR Impedance Bandwidth in table 2.1in the first entry.
5. I am always asked what I mean by the "narrow-bandedness" of a quarter-wave transformer: I find that I tend to reply: "It depends"...

   As an illustration of the this, repeat the exercise for a 250Ω load, (thus requiring a Z_0 of 111.8Ω), for a 500Ω load, and a 1000Ω load. Do this by simply clicking the `Select All` button, then the `Edit` button.

   Tabulate the answers in table 2.1.

   Generally, "impedance bandwidth" is defined as:

$$\text{(Upper frequency} - \text{Lower frequency) / Centre frequency} \qquad (2.4)$$

   so that for the first row of the table, $(492-109)/300 = 128\%$, or $300\pm64\%$, which is extremely wide indeed.

Table 2.1: Percentage bandwidth of a quarter-wave transformer for varying degrees of mismatch.

Load	Z_0 of $\lambda/4$ line	Low 2:1 Freq	High 2:1 Freq	Approx % BW
113	75	109	492	128
250	111.8			
500				
1000				21

Conclusion: As seen in table 2.1, the apparent "bandwidth" of a quarter-wave transformer depends on the degree of load mismatch. The reason for this apparent anomaly is apparent from the Smith Chart version of the VSWR plots you are making: this is seen in fig 2.7.

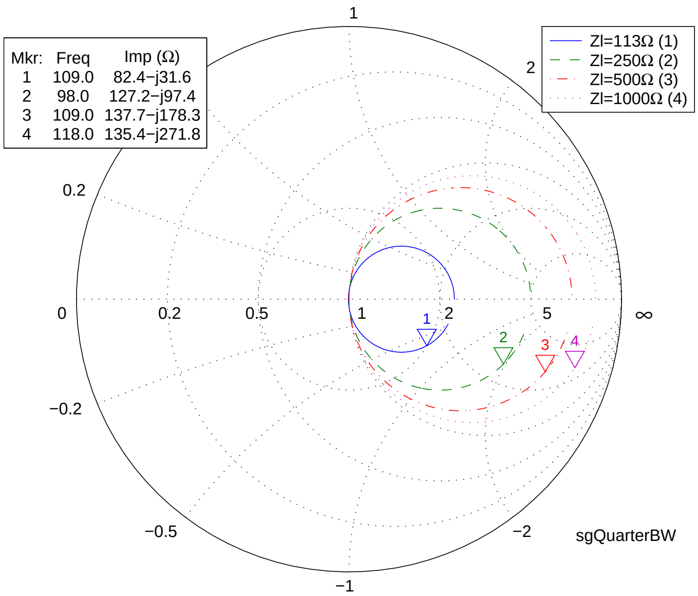

Figure 2.7: Different "bandwidths" of a quarter-wave transformer under differing (perfectly resistive) mismatch conditions.

Fig 2.7 shows perfectly how the purely resistive loads used thus far describe circle of constant resistance on the Smith Chart, thus giving different bandwidths depending on where they intersect the 2:1 constant VSWR circle.

Thus it is difficult to speak of the "bandwidth" of a quarter-wave transformer.

Exercise 2.2: Quarter-wave Transformer with dipole.

Purpose: *To illustrate the quarter-wave transformer with a "real" load.*

1. First simulate the dipole. Use `Add| Assembly| antennas| sndipole` and accept the defaults. Using `Edit| Simulation Settings` change the

Table 2.2: VSWR bandwidth of matched and unmatched dipole

	Low	High	Centre	Percent
Unmatched	274	294	284	7
Matched			285	

frequency to a sweep from `[50:400]`. Remember to change the model frequency in the Input Viewer to 400 MHz too. Plot the input impedance of the dipole on a Smith Chart.

2. Using the < button in the Input Interface to get `low` in the `Model Freq` box, select just the source segment of the dipole, and click the `Edit` button, and delete the excitation, using the `Delete` button in the `Excitation` section of the Dialogue box, *NOT* the `Delete` button of the main window.

   Add a segment to attach the transmission line to by `Add| Primitive| Segment`, changing the defaults so that `End1` is at `[0 1 0]` and `End2` is at [0 1 0.01]. Change the conductivity to 1/377, remembering that matlab `eval`'s anything you put into an input box, so don't calculate it: simply enter 1/377! Before closing the Dialogue box, add an excitation to the segment in the form of a default AFVS (Applied Field Voltage Source) of 1V (ie simply click `Add` in the Excitation part of the Dialogue).

   Click `Unselect All` then select the middle segment of the dipole and the new segment by using `Shift-Mouse1`. Use `Add| Primitive| Network| Transmission Line` to add a transmission line between these two segments.

   Use a Characteristic Impedance of 60Ω, Click on the `Set length to straight-line distance between segments` checkbox and fill in 0.25m instead. (Amazing what you can do with mathematical transforms: make a 1m transmission line 0.25m long!) Accept the default `linked` option.

   Overlay the results on the first impedance plot, and you will obtain something that looks like fig 2.8. Changing the format of the impedance plot to VSWR using `Format| VSWR`, it becomes clear that the dipole is better matched, and has a broader VSWR bandwidth. Using markers, calculate the improved VSWR bandwidth (assuming a 2:1 criterion) of the dipole as in table 2.2

Conclusion: Note that in fig 2.8, we see the effects not only of the dipole impedance changing with frequency, but also the effect that has on the transmission line, which itself is changing length as a function of frequency—so that the green dashed line in the figure actually represents quite a complicated set of transforms not easily visualised in any other way than SuperNEC visualisation.

Exercise 2.3: Quarter-wave Transformer with Folded Dipole

Purpose: *To illustrate the quarter-wave transformer using a more useful antenna: the folded dipole.*

Essentially, repeat the above exercise using a folded dipole.

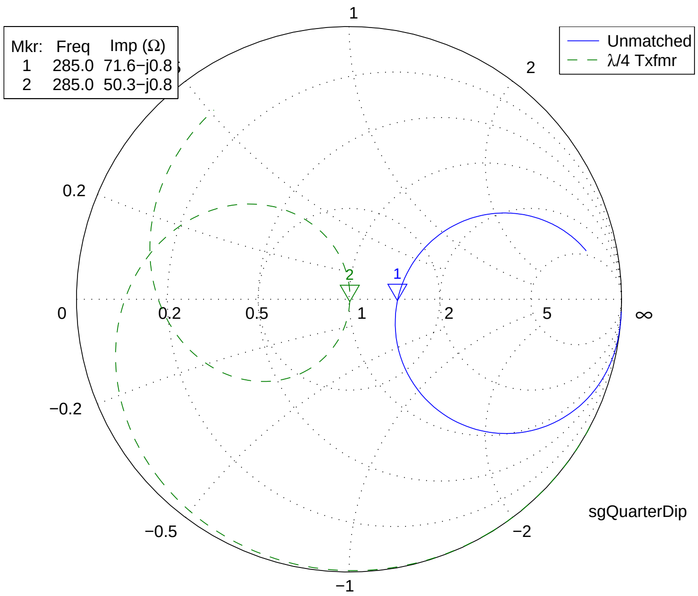

Figure 2.8: Unmatched dipole vs Quarter-wave transformer matching

1. Use `Add| Assembly| antennas| snfdipole` to add the folded dipole, changing the orientation to `[90 0 90]` and `Edit| Simulation Settings` to set the frequency to `[100:400]`, as before, and plot the input impedance on a Smith Chart. Record the impedance at resonance:

Resonant Impedance
$+j0\Omega$

2. Using the < button in the Input Interface, select `Group Level` of "low", and delete the excitation on the feed segment. Add a small segment using `Add| Primitive| Segment` at `End1 = [ 1 0 0]` and `End2 = [1 0 0.01]` of 1/377 conductivity with a 1V excitation. (Click the `Add` button in the `Excitation` sub-panel).
3. Assuming ideal conditions, calculate the characteristic impedance required to match the Resonant Impedance recorded above to 50Ω and install a quarter-wave transformer between the newly installed segment and the old feed segment of the folded dipole. ie Using `Shift-Mouse1` select the two segments, using `Add| Primitive| Network| Transmission Line` enter the characteristic impedance you have calculated, and click on `Set length to straight-line distance between segments` and enter 0.25m, and in response to the query box, ask the segments to be `linked`.
4. Simulate and plot the results which should be similar to fig 2.9

   Note however, that the *optimum* match has not been obtained for the antenna as a whole. This is simply because the "ideal" match was calculated for a single point only, not across a frequency sweep. Going back to the Input Viewer, use the < button to select the "low" `Group Level` and select the transmission line, and click the `Edit` button. Change the Characteristic impedance by 10Ω up and down and compare VSWR bandwidth

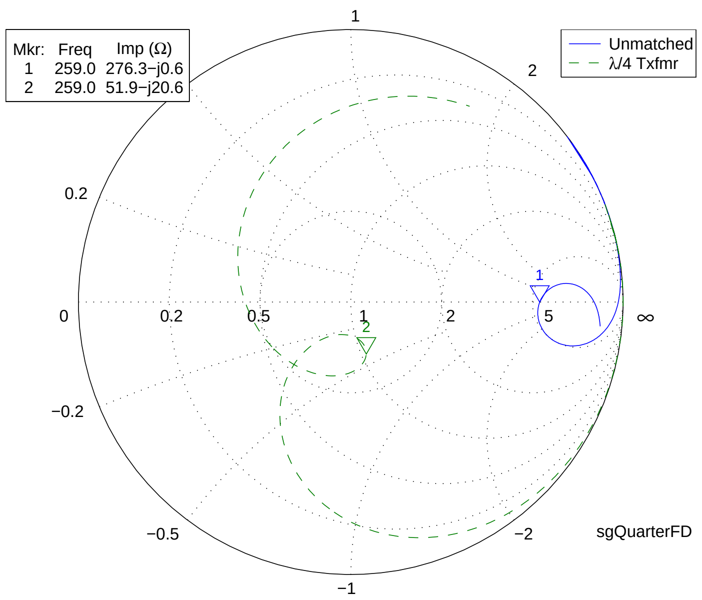

Figure 2.9: Unmatched Folded Dipole vs Quarter-wave Transformer Matching

Table 2.3: Table of VSWR bandwidths of a Folded Dipole under various Quarter-wave transformers

Z_0	Low	High	Centre	Percent
Default	226	301	259	28.9
			259	
			259	
			259	
			259	
			259	

results, recording them in table 2.3

My endeavours can be seen in fig 2.10, which shows that an "optimised" value of Z_0 can be obtained which spreads out the 2:1 VSWR over almost 45%.

Conclusion: The quarter-wave transformer is a very useful matching aid, but it must be remembered that an antenna is not just a single impedance point. The Folded Dipole is a much more broadband antenna than an ordinary dipole, this can be taken advantage of by an "optimised" Quarter-wave matching network which worsens the match in the centre (but still below 2:1) and broadens the match outside the centre.

Exercise 2.4: Multiple quarter-wave transformers

Purpose: *To illustrate the broad-banding effect of cascaded quarter-wave transformers for a thick dipole antenna.*

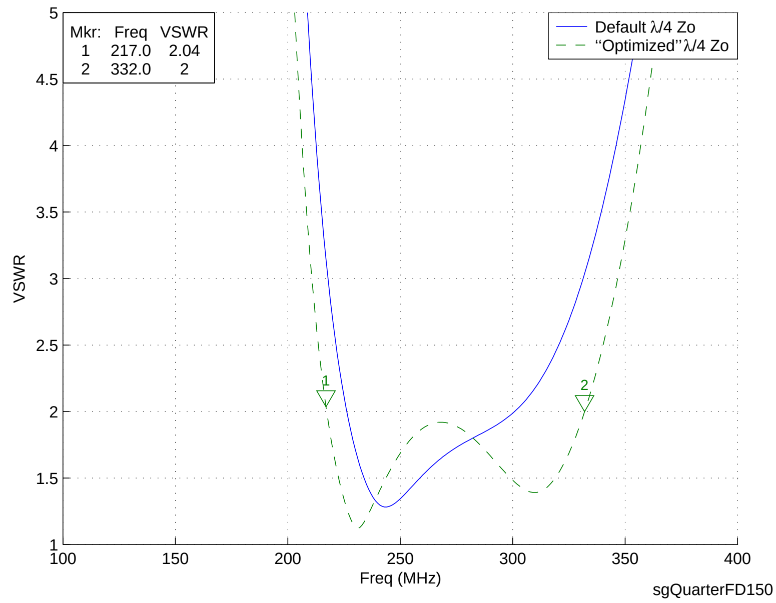

Figure 2.10: Comparison of the default $\lambda/4$ transformer and an "optimised" one.

Table 2.4: Resonant frequency and input impedance: SuperNEC vs Kraus

	Kraus Predicted	SuperNEC simulated
Resonant Frequency	276 MHz	
Input Impedance at Resonance	$65 + j0\Omega$	

In [Kraus, 1988, pg736], he shows a rather thick halfwave cylindrical dipole, matched via one quarter-wave transformer and two cascaded quarter-wave transformers. He shows something similar in [Kraus, 1984, pg421], but considers a resistive load, not a dipole.

1. Kraus uses a length L to diameter D ratio of 60, resulting in a Diameter of 0.00833m. Since SuperNEC needs radius, this becomes 0.00417m. Use `Add| Assembly| antennas| sndipole` to get the dipole, changing the radius.

   Using `Edit| Simulation Settings` setup a frequency sweep to be `[200:400]` MHz. (Remember to set the Model Frequency to 400MHz). Simulate and record the resonant frequency and input impedance in table 2.4.
2. Calculate the required characteristic impedance to match the above antenna to a 500Ω transmission line using equation 2.2. Back in the Input Viewer, set the `Group Level` to `low` via the $<$ button. Select the feed segment of the dipole, click the `Edit` button, and delete the excitation shown on it.
3. Via `Add| Primitive| Segment`, add a 0.01m vertical segment one metre away in the x direction, by setting the `End1` coordinates to `[1 0 0]` and

Table 2.5: Calculated characteristic impedances for the two-stage quarter-wave transformer

	Impedance
Z_{jn}	
Z_{01}	
Z_{02}	

`End2` to `[1 0 0.01]`, and the conductivity to 1/377. Also click the `Add` button of the Excitation segment of the dialogue box.

4. Click on `Unselect All`, then using the `Shift-Mouse1` click, select the middle segment of the dipole, and use `Add| Primitives| Network| Transmission Line` to add a transmission line of the characteristic impedance you have calculated from the above table.

   Click on `Set length to straight line distance between the segments` and enter a 0.25m transmission line instead.

   Request the they be `Linked` when asked.
5. Simulate and plot the impedance on a Smith chart. Set the Z_0 to 500Ω via `Options| Zo...`
6. Going back to the Input Interface, change the model to the double-cascaded transmission line as shown in fig 2.11

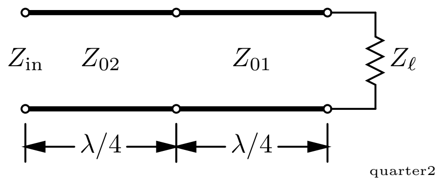

Figure 2.11: Double cascaded quarter-wave transmission line matching network between a 500Ω line and a half-wave dipole

The intermediate values of characteristic impedance are in a logarithmic relationship that correspond to binomial coefficients [Slater, 1942, pg60]. ie For a two-stage transformer, the logarithms are in the ratio 1:2:1, thus to get from 65Ω to 500Ω in two stages:

$$\ln\frac{108}{65} : \ln\frac{300}{108} : \ln\frac{500}{300} \approx 1:2:1 \tag{2.5}$$

It is easier to calculate the values using the geometric means:

$$\sqrt{500\times 65} = 180.3(Z_{\text{jn}}); \tag{2.6}$$

$$\therefore\ \sqrt{500\times 180.3} = 300.2(Z_{02}) \& \sqrt{65\times 180.3} = 108.3(Z_{01}) \tag{2.7}$$

Instead of the assumed 65Ω, use your resonant impedance recorded in table 2.4, and list the required characteristic impedances in table 2.5.

Thus: select the transmission line in the input viewer, click `Edit` and change its characteristic impedance to 108Ω (Actually use the value you calculated in table 2.5). Next, select the single segment, `Edit` it, and delete the Excitation.

Add a new segment to attach the second transmission line to by using `Add| Primitive| Segment` as before, entering the End coordinates as `[2 0 0]` and `[2 0 0.01]` and `Add` a standard excitation.

Click `Unselect All`, and then using `Shift-Mouse1` select the two segments and use `Add| Primitive| Network| Transmission Line` to add a 300Ω transmission line that is 0.25m long, giving something like fig 2.12

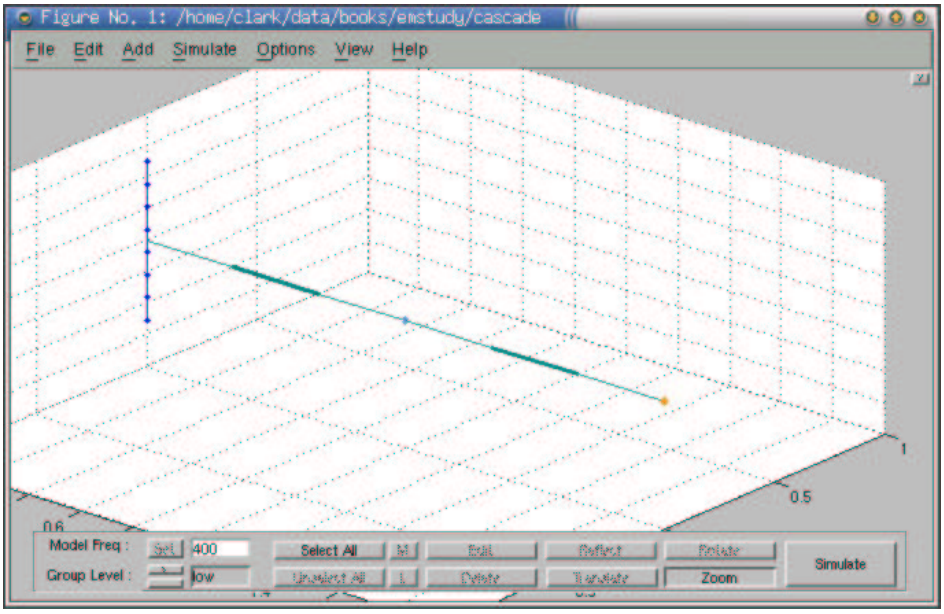

Figure 2.12: Two cascaded quarter-wave transformers

7. Simulate as before and overlay the impedance plots. Selecting the VSWR option from the `Format` menu, compare the bandwidths of the single and double cascaded quarter-wave lines.

   Fig 2.13 shows my results for a single, double, and triple quarter-wave lines.

Conclusion: Cascaded quarter-wave sections slightly increase the bandwidth available from the dipole. Since the dipole is inherently narrow-band, the effect is not that noticeable.

Exercise 2.5: Cascaded Quarter-wave Transformer with resistive load.

Purpose: *To illustrate the broadbanding effect of multiple quarter-wave transformers on a resistive load.*

This exercise shows the match of a 400Ω load to a 100Ω transmission line. The methodology should now be well known.

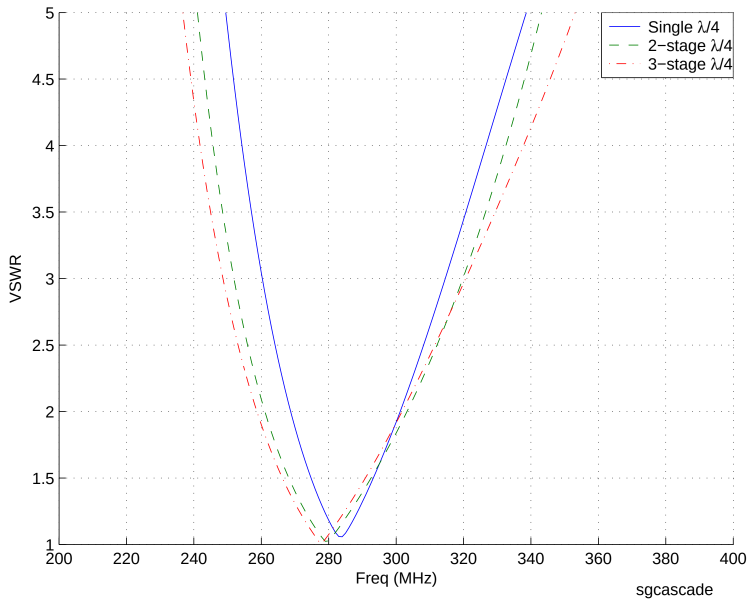

Figure 2.13: Single, double, and triple quarter-wave transformers, showing slight increase in bandwidth.

1. Using `Add| Primitive| Segment` changing the End2 coordinate to `[0 0 0.01]`; then add another with End1 at `[1 0 0]` and End2 at `[1 0 0.01]`, using `Add` to add a standard Excitation. Click `Unselect All`, then use `Shift-Mouse1` to select them both. It is important to select the segment without the source *first*.

   Using `Add| Primitive| Network| Transmission Line` add a 0.25m long 200Ω transmission line, the the `End1` admittance set to 1/400.

   Set the `Model Freq` to 500MHz, and setup a frequency sweep from 100 to 500 MHz, using `Edit| Simulation Settings`.

   Simulate and plot the VSWR, remembering to set the characteristic impedance to 100Ω using `Options| Zo...` in the impedance viewer.
2. Select only the feed segment, delete the excitation (before you add the next segment!) Add another segment at `[2 0 0]; [2 0 0.01]`, with a conductivity of 1/377, and a standard excitation. Select the last two segments and add a 0.25m 141.4Ω transmission line between them. Edit the other transmission line and change its characteristic impedance to 282.8Ω.

   Simulate and overlay the plot.
3. Repeat for a third section. The resultant plot should look something like fig 2.14.

Conclusion: As can be seen from fig 2.14, the effect of the narrow-band dipole has been removed and the broadbanding is purely due to the multiple quarter-wave stages used. Clearly, the effect certainly does widen the band over which it is effective, but note that the triple stage did not improve

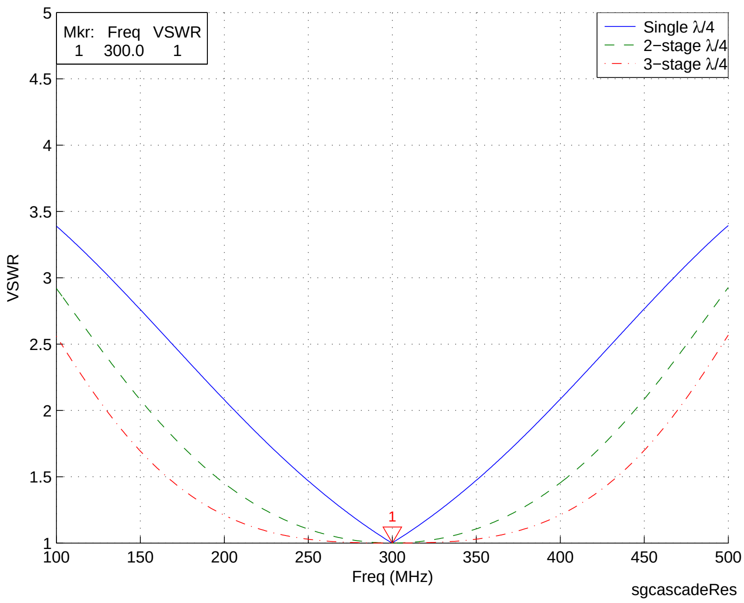

Figure 2.14: Bandwidth comparison of a single, double, and triple quarter-wave matching network with a purely resistive load.

over the double stage as much as the double did over the single. The Law of Diminishing Returns strikes again!

The concept can be taken further to an exponentially tapered transmission line, which of course, must be long enough, to match a very broad bandwidth.

Exercise 2.6: Power Splitter

Purpose: *To illustrate a power splitter, and to introduce the actual text output file of SuperNEC.*

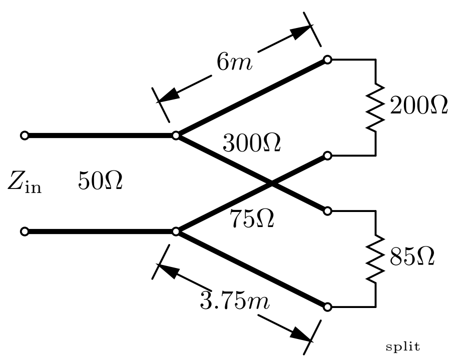

Figure 2.15: Power splitter

Fig 2.15 shows a classical matched power splitter. The 200Ω load is attached to the junction by a transmission line of a multiple of a half-wavelength. Hence, the characteristic impedance of the line is irrelevant at the centre frequency of 300MHz. The 85Ω load is attached to the junction by an odd multiple of a quarter-wave line. The combination at the junction results in a near perfect 50Ω match.

1. Create a simple three-segment splitter in a triangle, with the transmission lines between them as shown in fig 2.16

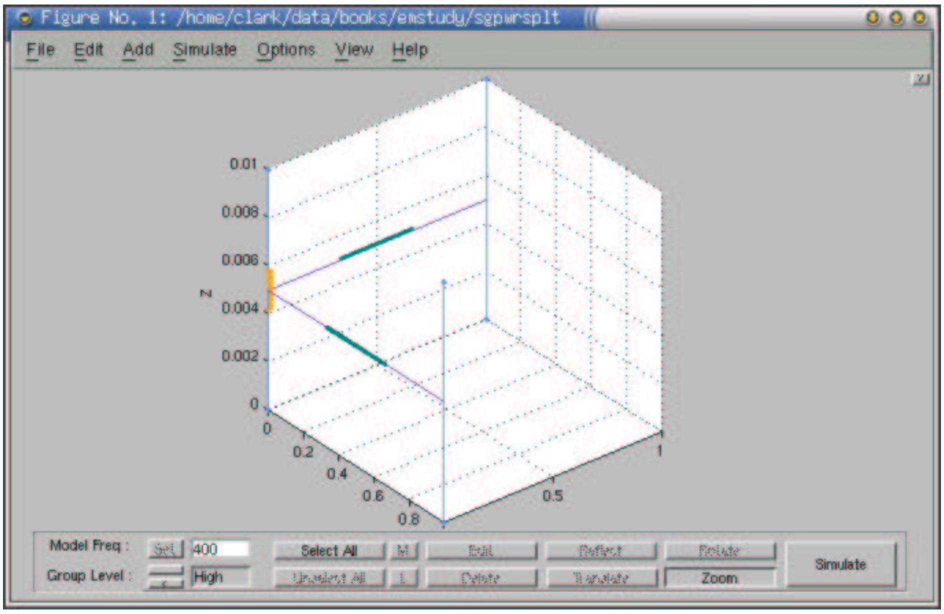

Figure 2.16: SuperNEC version of the Power splitter

The procedure should by now be extremely familiar:

- Using `Add| Primitive| Segment` add a segment with conductivity of 1/377 with a 1V source at `[0,0,0]; [0,0,0.01]`, and segments of conductivity 1/377 without sources at `[1,0,0]; [1,0,0.01]` and `[0,1,0]; [0,1,0.01]`; using a model Frequency of 400MHz.
- After `Unselect All`, select, using `Shift-Mouse1`, the segment with the source and one other, selecting the source ***first***. Attach a transmission line of the appropriate characteristic impedance, and fill in the appropriate `End2` admittance as 1/200 or 1/85 as appropriate.
- Add a frequency sweep from 200 to 400 MHz, and plot the VSWR which should look like fig 2.17.

2. Re-edit the Simulation settings and specify a single frequency of 300MHz. Simulate.
3. From the matlab command window edit the SuperNEC output file. Assuming you have saved your power splitter structure as `sgpwrsplit`, then the command `edit sgpwrsplt.out` will bring up the output file. Search for the string `- - - ANTENNA INPUT PARAMETERS - - -`.

The input parameters are those associated with the only source used in the simulation, and gives the total input power at the very end of that

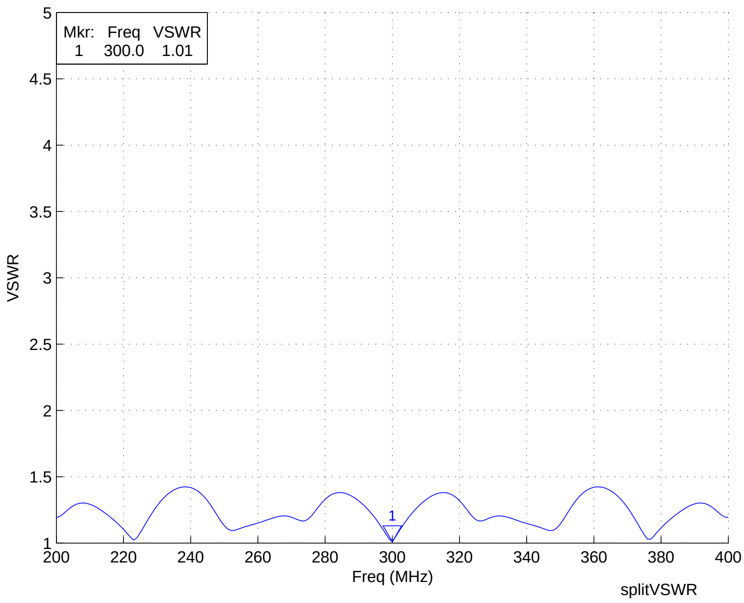

Figure 2.17: VSWR of the matched power splitter

Table 2.6: Calculated versus Simulated power

	Calculated		Simulated	
	watts	% of P_T	watts	% of P_T
P_T				
P_{200}				
P_{85}				

line. Record that number in table 2.6.

Just above the Input Parameters, you will see a section entitled Structure Excitation Data at Network Connection Points, which lists data at each end of each transmission line. ie There should be 4 lines of data: we are interested only in the non-negative power quantities. Record these in table 2.6, calculate the percentages and compare to your hand-calculated values.

Conclusion: SuperNEC correctly predicts the power split between the two loads. The output file from SuperNEC contains a lot more information than is apparent from the GUI Output Viewer, and this information is often useful.

Exercise 2.7: Single Stub-match

Purpose: *To illustrate the single stub-match, with iterative visualisation.*

In stub matching examples, there are only two key things to remember:

1. A short-circuited stub cannot provide any real impedance or admittance, *its input impedance is purely imaginary.*

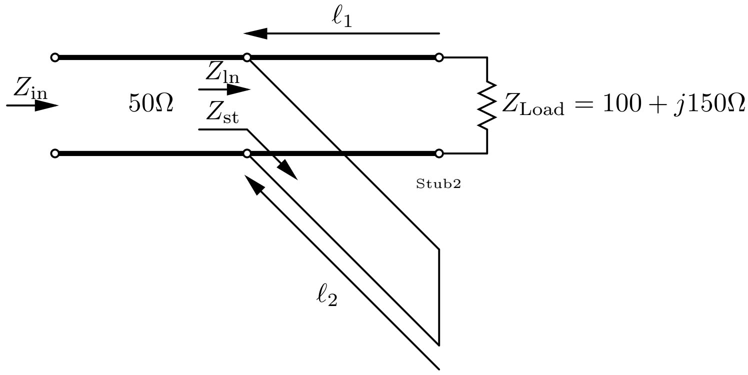

Figure 2.18: Stub matching example.

2. The goal is to be matched on the left hand side of fig 2.18.

Thus, if the goal is to be matched on the left hand side of fig 2.18, then it is clear that we have to be matched at the junction! If we have to be matched at the junction, *and* the stub can only provide *imaginary* impedance, then the real part of the impedance at the junction before the stub is attached must equal the real part of the desired match.

In the above example, where all the transmission lines have a characteristic impedance of 50Ω, I therefore want $50 + j0\Omega$ at the junction after the stub has been connected. Since the stub can only provide imaginary impedance, before it is connected to the junction, the real part of the impedance on the line at that point must be 50Ω. ie $\Re Z_{\text{ln}} = 50\Omega$.

- you can ***move, or "walk"*** down a transmission line: ie *move* a *distance* whilst being confined to a constant VSWR circle (ie a circle centred at the centre of the Smith Chart), and
- you can add reactive impedance/admittance using a stub, causing the resultant impedance to travel along a path of constant resistance, but this is not a *distance moved*.

1. Since we don't need the transmission line system to the left of the junction, set up three short segments as before, with the two transmission lines linking them.

   Using `Add| Primitive| Segment` add a segment with conductivity of 1/377 with a 1V source at `[0,0,0]; [0,0,0.01]`, and segments of conductivity 1/377 without sources at `[1,0,0]; [1,0,0.01]` and `[0,1,0]; [0,1,0.01]`; using a model Frequency of 300MHz.

   After clicking `Unselect All`, select the source segment first, and the load segment next, and use `Add| Primitive| Network| Transmission Line` add a default 50Ω transmission line with a `Load End 2 (admittance)` of $1/(75 + 25j)$, of length 0.001m.

   After clicking `Unselect All`, select the source segment first, and the stub segment next, and use `Add| Primitive| Network| Transmission Line` add a default 50Ω transmission line with a `Load End 2 (admittance)` of

1000000, of length 0.25m. (ie a short circuit $\lambda/4$ away = an open circuit at the junction!)

2. Simulate, and plot the impedance. Using `Format| Smith Chart| Admittance` change the Smith Chart view to admittances. (Recall that, since we are working with parallel impedances, it is far easier to work in admittances).
3. Click on the transmission line between the source and the load and `Edit` it and lengthen it to 0.065m. Overlay the admittance plot. Repeat for 0.129, 0.194m (If you have done the stub match manually on a Smith Chart, you should recognise the last number!)
4. Now that we are on the R=G=1 circle, the *real* part of the admittance Y_{ln} is equal to the required value for matching, and all that needs to happen is that the stub must provide the imaginary part of the opposite value.

   As seen from fig 2.19 the admittance at the R=G=1 circle has a positive imaginary part. Thus, negative imaginary part must be provided by the stub: ie inductive susceptance (= capacitive reactance).

   Click on the stub transmission line and change it from 0.25 to 0.2 and then 0.167m. The result of this manipulation is shown in fig 2.19

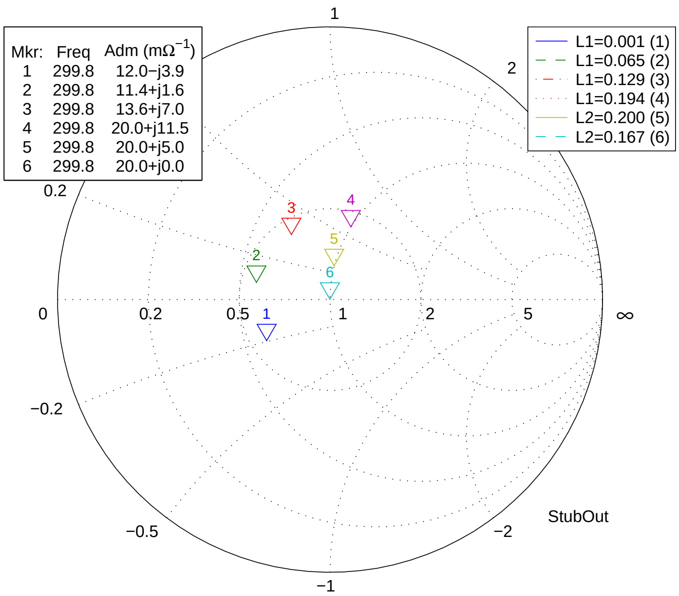

Figure 2.19: Admittance Smith Chart progression for a single stub-match

Conclusion: As seen in fig 2.19, changing L1, or "walking" down the transmission line results in the point walking along a constant VSWR circle. When the R=G=1 circle is contacted, the stub length is changed to cancel out the imaginary component. Note that points 4,5,6 in fig 2.19 all have the same real part—varying the stub cannot change the real part!

A very good illustration of the Stub-matching technique is found in fig 2.20, which shows a stub-match in various stages being applied to a Folded Dipole. The line associated with Marker 1 shows the Folded Dipole admittance without any matching. It is clearly seen that it is very far away from being well-matched to the 50Ω system!

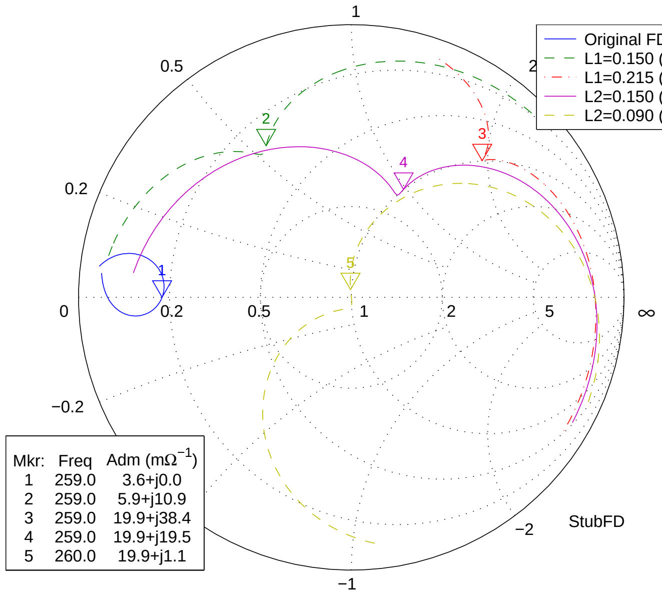

Figure 2.20: Stub-match applied to a Folded Dipole (Smith Chart is in admittances)

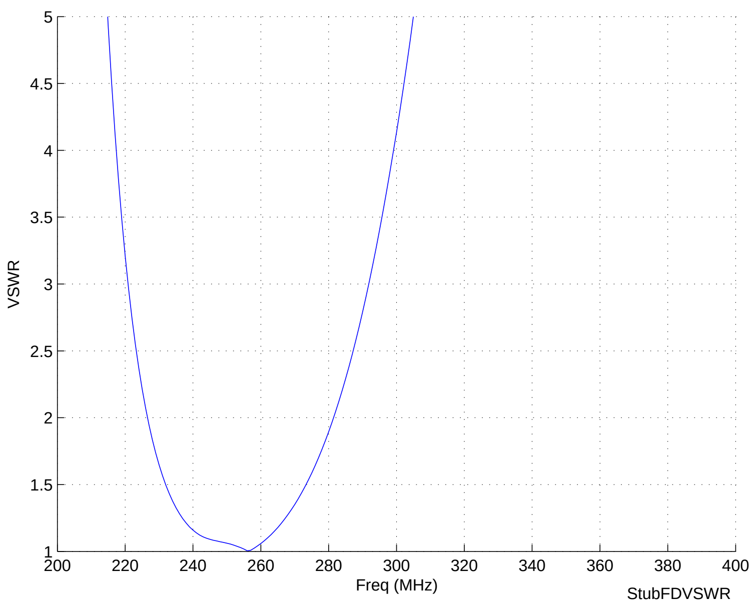

Figure 2.21: VSWR of the Stub-matched Folded Dipole

Marker 2 shows the admittance after a 0.150m 50Ω line has been attached, and Marker 3 at 0.215m at which point the R=G=1 line has been reached at the desired frequency. Now adding a short circuited stub to cancel the large positive susceptance component, Marker 4 shows what has happened with a 0.150m stub, and finally Marker 5 shows the result with a 0.090m stub. It can thus be seen that the stub matching technique is very useful for real-world antennas! The VSWR is shown in fig 2.21.

2.3 Problems

2-1. Why Match? Derive, from first principles, equation 2.1.

2-2. Quarter-wave transformer Using SuperNEC, show that a quarter-wave transformer of any characteristic impedance converts an open circuit to a short circuit, and vice versa.

2-3. Quarter-wave transformer Simulate a Yagi-Uda to obtain its resonant impedance, and apply a quarter wave transformer to it following the SuperNEC methods used in this chapter. Plot the impedance bandwidth.

2-4. Quarter-wave transformer Repeat the previous exercise, but change the characteristic impedance of the quarter-wave transformer by a few ohms up and down in order to optimise the 2:1 VSWR bandwidth, as in exercise 2.2.

2-5. Multiple Quarter-wave transformers Repeat exercise 2.2, but calculate the necessary characteristic impedances for a four and five stage cascaded quarter-wave matching section. Compare the percentage increases of the bandwidth that you get for each additional stage.

2-6. Power Splitter Use the SuperNEC `sndipole` assembly to place three vertical dipoles in a row along the x-axis, with a half-wavelength between them. Simulate to obtain the resonant impedances (due to mutual coupling, the middle dipole's input impedance will be different). Plot the radiation pattern in the xy plane.

Design a power splitter using three transmission lines to provide twice as much power to the middle dipole than that given to each of the outer dipoles. After deleting the excitations on the dipoles, add a small segment and link it to all three feed segments with transmission lines of your calculated parameters. Plot the radiation pattern again. (See chapter 5).

2-7. Stub Matching Replicate the Folded Dipole Stub match shown in figures 2.20 and 2.21 by the usual technique of deleting the excitation, adding tiny segments and attaching transmission lines to them.

2-8. Stub Matching Perform a Stub Match on a Yagi antenna.

2-9. Double Stub-match Another stub-matching technique is the double-stub: instead of varying the length from the load to the stub, two stubs are placed at fixed distances from the load, and only the lengths of the two stubs are varied. This technique is often employed in microwave waveguide systems where varying the distance from the load could involve a lot of plumbing work! Moving a shorting plate in a waveguide stub is much easier.

Traditionally, the first stub is $\lambda/4$ from the load and the second stub is $\lambda/8$ further on. Since we need to be on the R=G=1 circle at the second stub, that implies that by the first stub we must contrive to be on a circle shifted $\lambda/8 = 90°$ towards the load.

Repeat example 2.2, but using a double stub of your design. Compare the bandwidth obtained by the double stub with that obtained by the single stub.

2-10. Creating Assemblies Create an assembly which attaches a standard Stub-match to a Yagi antenna, automating the process of attaching the transmission lines, and making their length more easily configurable.

2-11. Creating Assemblies Create an assembly that allows multiple quarter-wave transformers to be automatically created in cascade.

2-12. Creating Assemblies Create an assembly that easily creates a double-stub tuner.

Chapter 3

Waves

This chapter examines Electromagetic Waves in Space.

3.1 Theory

An EM wave travels in free-space *and in most transmission lines* as a Transverse EM wave (TEM). This implies that the direction of propagation is at 90° to both the Electric and Magnetic wave, which, in turn are at 90°, as shown in fig 3.1

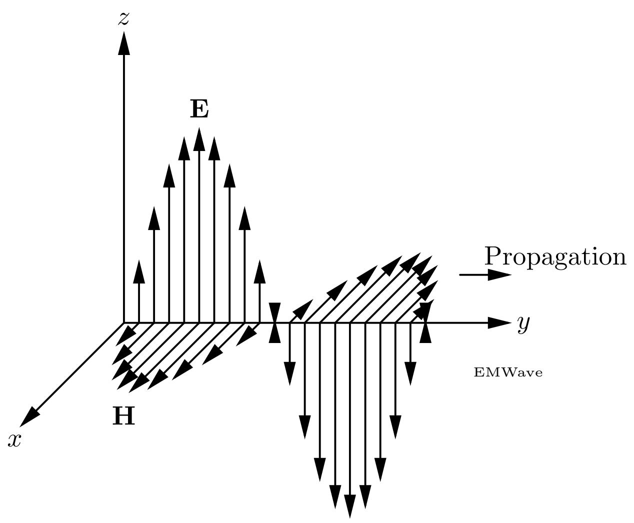

Figure 3.1: Transverse Electromagnetic Wave in free-space

Since the **E** field is analogous to voltage and the **H** field to current in the circuits sense, it is easily seen that the equivalent relationships to Ohms law etc exist in TEM waves in free-space:

$$(V = I \times R) \therefore \mathbf{E} = \mathbf{H} \times \eta \quad \text{or:} \quad \mathbf{H} = \frac{\mathbf{E}}{120\pi} \tag{3.1}$$

In a similar fashion, the power relationships hold (power density in EM):

$$S = \mathbf{EH} = \frac{\mathbf{E}^2}{120\pi} = \mathbf{H}^2 \times 120\pi \quad \mathrm{W/m^2} \tag{3.2}$$

where $\mathbf{E}$ and $\mathbf{H}$ are RMS values.

In general, EM waves of a frequency above 30MHz do not bend around the earth, and propagation occurs only within Line-of-Sight (LOS).

3.1.1 Reflection from the Earth's Surface

In addition to its role as a obstacle, the earth's surface also acts as a *reflector* of radio waves. This situation is illustrated in figure 3.2.

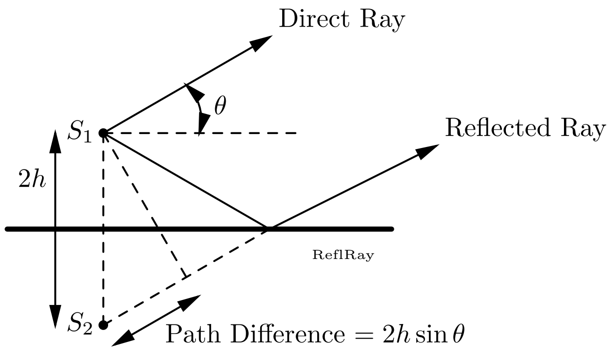

Figure 3.2: Geometry of Interference between Direct Path and Reflected Waves

It is clear that if S_1 is an isotropic source and would normally radiate equally well in all directions, the pattern would be modified by the reflected wave. By the method of images the situation above is similar to that which exists if a mirror image source S_2 was positioned at distance h below the reflecting plane. Clearly there will now be a difference in the path lengths to some distant point P. At certain elevation angles θ the path difference would be such that the two waves are in phase and thus interfere constructively and for others the interference would be destructive and result in a null in the radiation pattern. If the field due to a single source is termed E_0 then the total field would then be given by:

$$\mathbf{E} = |E_0| \sin\left(\frac{2\pi h \sin\theta}{\lambda}\right) \tag{3.3}$$

This condition is not always advantageous since an antenna that may have had a maximum towards $\theta = 0°$ would now have a null in the same direction. The only way to improve the situation would be to either make the antenna higher and thus force the angle of the first maximum lower or increase the frequency and thus ensure an increased h/λ ratio.

In cases where radiation at some angle is required this reflection results in an unexpected bonus, however. The maximum value of the **E**-field in the direction of the maxima is twice the value of the original antenna without reflection. This implies that in that particular direction the power density would be increased by a factor of four (power density is proportional to the square of the **E** field). Earth reflection can thus be used to gain a 6 dB bonus in antenna gain if used properly!

3.2 Exercises

Exercise 3.1: Ground Reflections

Purpose: *To illustrate the Reflection from the Ground, and to highlight the fact that you can't get away from earth!*

1. Place a horizontal dipole at a height of 1.44λ at 300MHz, by `Add| Assembly| antennas| sndipole` and modify the `Location` to `[0 0 1.44]` and the `Orientation` to `[0 90 0]`.
2. `Add| Ground` and choose a `Perfect` ground `Type`.
3. `Edit| Simulation Settings` and add a 2D radiation pattern in the xz plane, remembering to change the `Theta` entry from `[0,360,361]` to `[-180,180,361]` as usual.

   The radiation pattern obtained is shown in fig 3.3.

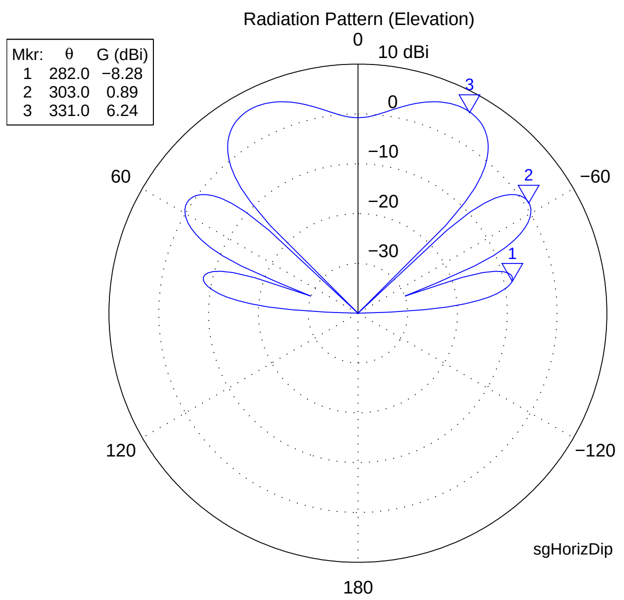

Figure 3.3: Radiation Pattern of a Horizontal Dipole 1.44λ above a perfectly conducting, infinite ground.

4. Select the dipole and click `Edit` and vary the height to see the effect of the ground plane as a function of height.

Conclusion: The presence of the ground plane strongly affects the radiation characteristics of any antenna. There is no way to get rid of its effects, but after a certain height, the effect is dimished. Naturally, the effect is less pronounced when a vertical antenna is used.

Exercise 3.2: Shielding effectiveness

Purpose: *To illustrate the shielding effect of a wire mesh.*

1. Create a very small dipole using `Add| Assmebly| antennas| sndipole` editing the `End1,2` z coordinates to be 0.05 and −0.05 to make a very short dipole.
2. `Add| Assembly| structures| snbox` creates a gridded box. Change the defaults so that the `Location` is at `[0 0 -0.25]`, the `Length, Width, Height` all at 0.5m, and the `Freq. Scaling` at 0.1, to provide a 0.5m cube of only one segment at the vertices as shown in fig 3.4.

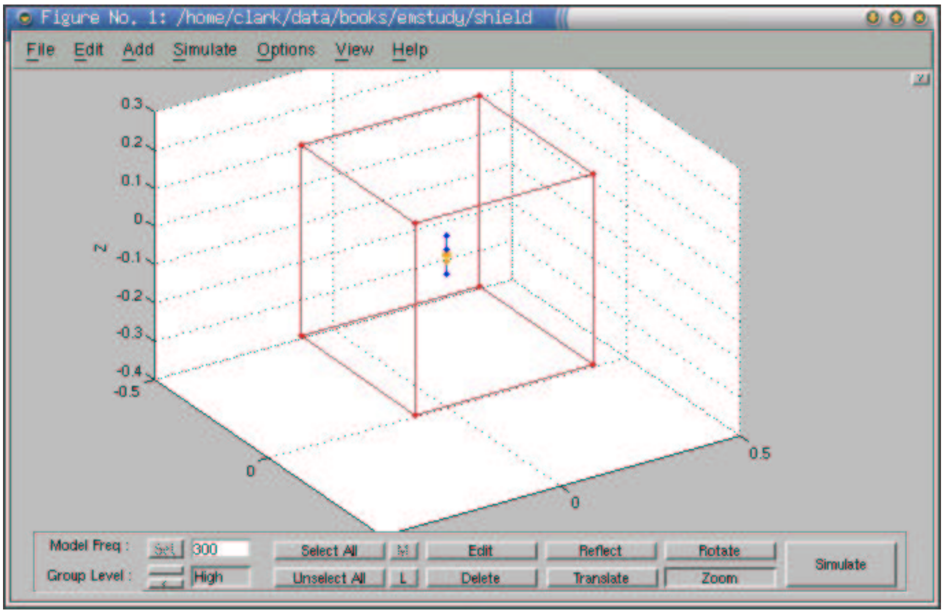

Figure 3.4: Short dipole inside an undersegmented box.

3. Using `Edit| Simulation Settings`, add an xy plane Radiation Pattern, and simulate. The result is shown in fig 3.5
4. Going back to the input viewer, select the box only, and click `Edit`. Change the `Freq. Scaling` to 0.5 to get a box with more segments. Plot.
5. Change the `Freq. Scaling` to 0.9, and you wil get a nice, uniform -999dB plot!!!

Conclusion: If you make a wire grid structure, where the spacing between the grid elements is about a tenth of a wavelength, it appears as if it was a solid metal sheet. No radiation penetrates it.

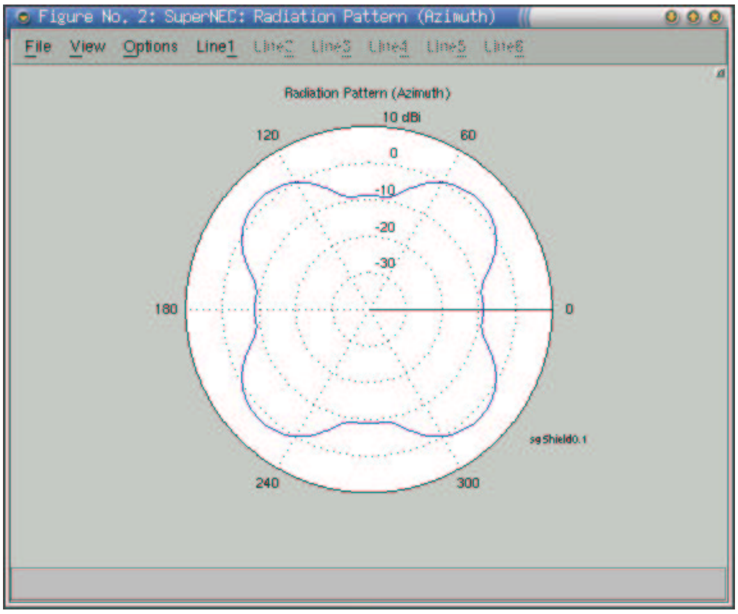

Figure 3.5: Shielding due to a 0.1 undersegmented box.

3.3 Problems

3-1. Ground Interaction Repeat exercise 3.2 using a vertical dipole.

3-2. Ground Interaction Repeat exercise 3.2 using a finitely conducting ground (Use the defaults) Note that the use of a Perfect Ground produces very harsh interactions which do not occur as severely in reality.

Chapter 4

Basic Antennas

This chapter examines the characteristics of the basic antenna building blocks, namely: dipoles, loops and monopoles.

4.1 Theory

4.1.1 Ideal Dipole

The ideal dipole must be one of the most useful theoretical antennas to understand as a large number of other antennas are analyzed using the equations that are quite easily developed for this antenna. Examples of these are the short dipole, loop antennas, travelling wave antennas and some arrays. The radiation pattern of any wire construction on which the currents are known can also be readily determined by considering the structure to consist of connected ideal dipoles and adding the pattern contribution due to each to form the full pattern. Many computer analysis codes rely on this approach.

The ideal dipole is defined as a linear wire antenna with length very small with respect to the wavelength and a *uniform current distribution.* For convenience, this antenna is positioned at the centre of the coordinate system and aligned in the z-direction, as shown in figure 4.1.

4.1.2 Fields

Using Maxwell's equations and the simplicity of this geometry it is very easy to find the fields due to the constant current I [Kraus and Fleisch, 1999, pg278]. When such an analysis is performed it is found that the far field of the antenna has an **E**-field in the θ direction, E_θ, and a ϕ-directed **H**-field, H_ϕ only. The expression for the E-field will be given but the H-field can clearly be found by "Ohm's Law of Free Space" as discussed in section 3.1.

$$E_\theta = \frac{60\pi I_0 \ell}{\lambda r} j e^{j(2\pi f - \beta r)} \sin\theta \tag{4.1}$$

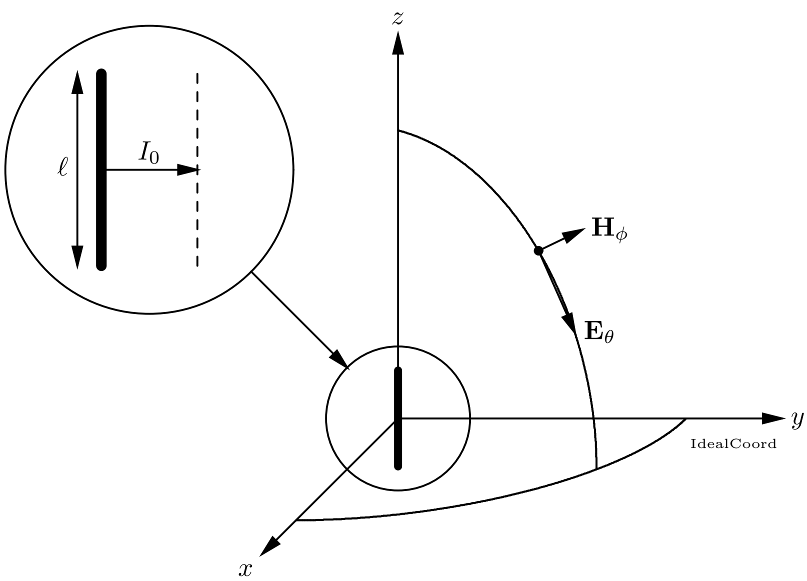

Figure 4.1: The Ideal Dipole in Relation to the Coordinate System

There are a number of important points relating to this expression. Considering it factor by factor:

- 60π is the constant or magnitude
- I_o is the (constant) current magnitude. An increase in this value results in a corresponding increase in the field
- $\frac{\ell}{lambda}$ is the *electrical length* of the antenna and again an increase in this ratio will imply a larger field. Changes in this ratio should only be made such that the assumption of small electrical length still holds (0.1λ maximum).
- $je^{j(2\pi f-\beta r)}$ is the phase factor. This factor is relatively unimportant unless this antenna is combined with another and the total pattern becomes an addition of the fields where phase plays an important role.
- $\sin\theta$ is the pattern factor. This is the only factor indicating variation with respect to the spherical coordinate system angles. Since none of the factors contain a ϕ-term this antenna has constant pattern characteristics in the azimuth direction. The resulting pattern has the familiar "doughnut" shape as illustrated in figure 4.2

Another way of putting this is that the antenna has *omnidirectional azimuthal coverage.*

The form of equation (4.1) is common to the expressions for most antenna field distributions. Such distributions are always a function of excitation, geometry in terms of wavelength and θ and ϕ angles. The relative pattern of the antenna can be drawn using only the $\sin\theta$ term and regarding the rest as a normalizing factor. Where absolute field strengths are required the total equation should clearly be used.

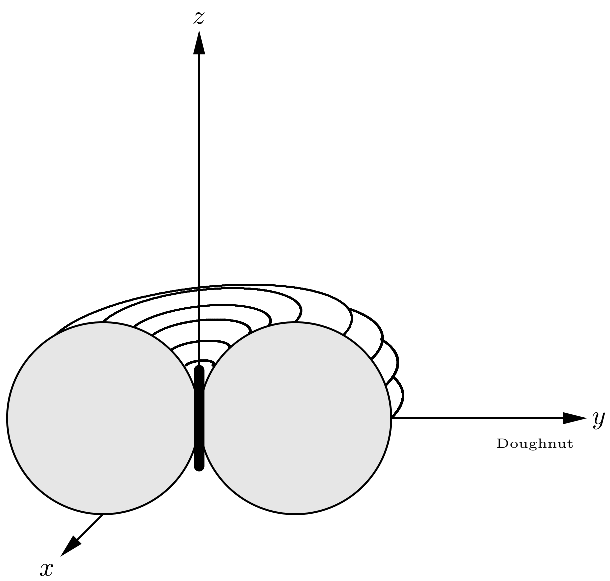

Figure 4.2: Pattern of an Ideal Dipole Antenna

4.1.3 Radiation resistance

The radiation resistance of the antenna can be found once the field distribution is known. Using circuit concepts, the radiation resistance R_r is given by:

$$R_r = \frac{2P_t}{I_0^2} \quad \Omega \tag{4.2}$$

The total power transmitted P_t is found by integrating (adding) the power density over a surface surrounding the antenna. Clearly if the power densities in all directions have been accounted for, the total power is found. The power density in any direction can be found using the expression discussed before:

The factor of two is introduced as result of the fact that I_0 is the peak current and not the RMS value.

$$P_d = \frac{E^2}{2(120\pi)} \tag{4.3}$$

Performing this integration, an expression for total power radiated is obtained and using 4.2 the radiation resistance is found as:

$$\boxed{R_r = 80\pi^2 \left(\frac{\ell}{\lambda}\right)^2 \quad \Omega} \tag{4.4}$$

This value is clearly always small since the ratio of antenna length to wavelength (ℓ/λ) was assumed to be small ($\leq$0.1) at the outset of the analysis.

4.1.4 Directivity

The directivity of the ideal dipole is calculated by assuming an input power of 1 W to the antenna. Since the reference used is always the isotrope, the power that it would radiate, given the same input power is simply $P_{d\ \text{(isotrope)}} = \frac{1}{4\pi r^2}$.

The current to an ideal dipole with 1 W input power is given by $I_0 = \sqrt{\frac{2}{R_r}}$. Using (4.4) for R_r in the expression above results in:

$$I_0 = \sqrt{\frac{2}{80\pi^2(\ell/\lambda)^2}} \tag{4.5}$$

Substituting (4.5) into (4.1) the E-field can be found in the maximum direction ($\theta = 90°$). The power density in this direction, P_d (ideal dipole) is found by the relationship:

$$\begin{aligned} P_d &= \frac{E^2}{2(120\pi)} \\ &= \frac{(60\pi)^2\ 2\ \ell^2}{(\lambda r)^2\ 80\pi^2\ (\ell/\lambda)^2\ 2(377)} \end{aligned} \tag{4.6}$$

The directivity by definition is the ratio, which becomes:

$$\boxed{D = \frac{P_{d\ \text{(ideal dipole)}}}{P_{d\ \text{(isotrope)}}} = 1.5 (= 1.76\text{dBi})} \tag{4.7}$$

4.1.5 Concept of current moment

An important concept which allows the use of the results achieved for the ideal dipole above to other antennas is that of *current moment*. By inspection of (4.1) it is clear that the E-field is proportional to the product of the length of the antenna and the current (assumed constant over the whole antenna). The current moment M for an ideal dipole is therefore the *area* under the current distribution:

$$\boxed{M = I_0 \ell} \tag{4.8}$$

The power density and power transmitted is proportional to the current moment *squared* — ie:

$$\begin{aligned} E &\propto M \\ P &\propto M^2 \end{aligned} \tag{4.9}$$

4.2 The Short Dipole

dipole The short dipole antenna is a practically realizable antenna which is assumed to have a *triangular* current distribution when shorter than about a tenth of a wavelength, as shown in figure 4.3.

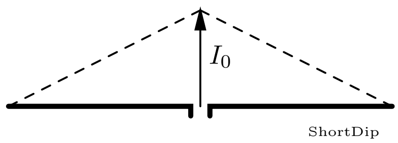

Figure 4.3: Current Distribution on a Short Dipole Antenna

Since it can be shown that the current distribution on thin linear radiators is sinusoidal, the two small parts of a sinusoid starting at either tip of the short dipole is well approximated by two straight lines and hence a triangular distribution.

4.2.1 Fields

The current moment of the short dipole in terms of the feedpoint current I_{in} is:

$$M = \frac{I_{\text{in}}\ell}{2} \tag{4.10}$$

The E-field from the antenna is thus half the E-field found for the ideal dipole (disregarding the phase terms which would be the same) i.e.

$$\boxed{E = \frac{30\pi I_{\text{in}}\ell}{\lambda r}\sin\theta} \tag{4.11}$$

4.2.2 Radiation resistance

The power transmitted by the short dipole is proportional to the *square* of the current moment (ie a quarter):

$$P_t(\text{short dipole}) = \frac{\mathrm{P_t}(\text{ideal dipole})}{4} \tag{4.12}$$

since $P_t = I^2R$ the radiation resistance of the short dipole would be a quarter of that of the ideal dipole

$$\boxed{R_r(\text{short dipole}) = 20\pi^2\left(\frac{\ell}{\lambda}\right)^2 \quad \Omega} \tag{4.13}$$

4.2.3 Reactance

The reactance of a short dipole is always *capacitive* and usually quite large and is not as easily calculated as the radiation resistance. Reactance values can be measured for a specific antenna—and tables King and Harrison [1969]

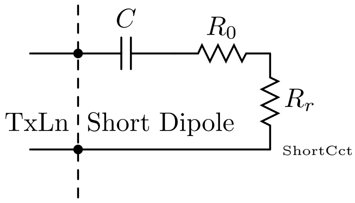

Figure 4.4: The equivalent circuit of a short dipole antenna

are available for different thickness antennas. The equivalent circuit of a short dipole antenna can be given as in figure 4.4

The R_0 value indicated in figure 4.4 refers to the *loss resistance* and should be included when that value is significant in relation to the radiation resistance R_r.

This antenna thus presents a serious problem when power has to be delivered to it. The capacitive reactance ($X = -1/2\pi fC$) is typically a few hundred ohms which is a large mismatch condition. Matching is usually accomplished by placing an inductor in series with the feed line which has a positive reactance ($X = 2\pi fL$) that is equal in magnitude to the capacitive reactance thus resonating the antenna, as shown in figure 4.5.

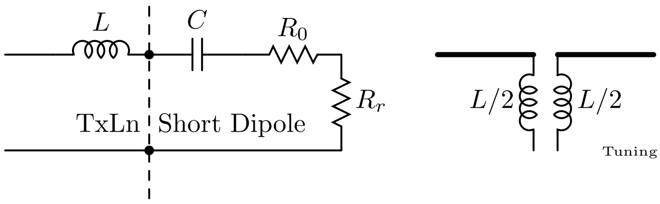

Figure 4.5: Tuning out dipole capacitive reactance with series inductance

This is an improvement but a few “catch-22” problems still exist which explains the inherent difficulty in transferring power to small antennas:

- The coil will have some loss resistance which is very often large compared to radiation resistance (which is often a fraction of an ohm) resulting in very low efficiency.

- To decrease coil losses the Q of the inductor should be increased but this causes a reduced operating bandwidth and a more sensitive antenna, also increasing the circulating currents and hence the voltages associated with them.

- If the decrease in bandwidth can be tolerated, the resultant real (resonant) impedance would approximate the very low radiation resistance and this still presents a matching problem.

4.2.4 Directivity

It is clear from the $\sin\theta$ factor in the E-field expression that the shape of the pattern is exactly the same as that of the ideal dipole. The directivity (gain) of a short dipole is therefore equal to the gain of the ideal dipole:

$$\boxed{D \text{ (short dipole)} = 1.5} \tag{4.14}$$

4.3 The Short Monopole

monopole When a ground plane is present as in figure 4.6 antennas can be

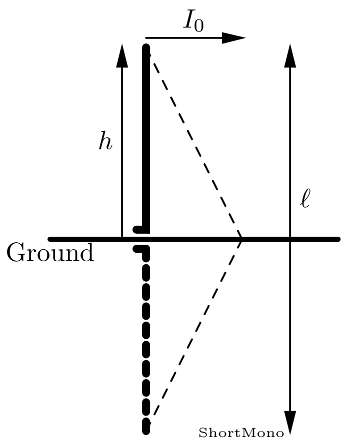

Figure 4.6: Short monopole antenna

analyzed in terms of *image theory*. The antenna/image combination has the same radiation pattern as the short dipole. The two major differences between the two are:

- the monopole current moment is half that of the dipole
- the monopole radiates no power in the lower hemisphere—for the same input power as the dipole, the monopole radiates twice as much power into the upper hemisphere.

Once image theory is applied (and this is true of any antenna/image combination) the ground-plane behaviour can be deduced from that of the free space equivalent.

The power radiated is halved and the radiation resistance is half that of a short dipole when expressed in terms of ℓ. For monopoles, the length of the antenna above the ground $h = \ell/2$ is clearly more relevant than ℓ and in terms of this the radiation resistance is:

$$\boxed{R_r = 40\pi^2 \left(\frac{h}{\lambda}\right)^2} \tag{4.15}$$

All the power is radiated in the upper hemisphere which implies double power density in all directions in comparison to short—or ideal dipoles with the same power input. The directivity of this antenna would thus also be double that of the previous two antennas:

As usual, increased gain is at the expense of decreased gain elsewhere—under the ground plane in this case!

$$\boxed{D \text{ (short monopole)} = 2\ (1.5) = 3} \tag{4.16}$$

4.3.1 Input impedance

It was shown above that the radiation resistance of the short monopole is half that of the equivalent short dipole. The same applies to the capacitive reactance of the antenna.

4.4 The Half Wave Dipole

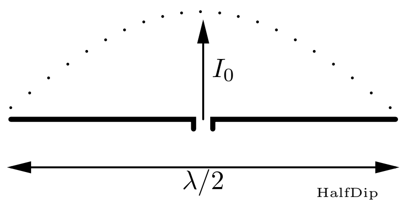

Figure 4.7: A Half wave dipole and its assumed current distribution

Although the derivation will not be performed here, the fields from a half wave dipole with an assumed sinusoidal current distribution as shown in figure 4.7 can also be found by considering the antenna to be made up of small ideal dipoles. The only difference in this case is that the phase of the current can not be assumed to be constant and that the path lengths to a distant point P can differ from the different locations on the antenna.

It is interesting to note that the current distribution must be known before the various parameters of an antenna can be determined.

In the above cases, the current distributions were assumed to be sinusoidal making analysis possible. This assumption is quite valid for thin linear radiators as was shown by Schelkunoff [1941] and others. For more complex structures (and thick dipoles) the current distribution may be more difficult to determine. Computational techniques such as the *Method of Moments*, embodied in SUPERNEC, are therefore primarily concerned with the determination of the current on the antenna wires. Once this is known it is a relatively straightforward task to calculate impedance and radiation pattern of the antenna.

4.4.1 Radiation pattern

Using the sinusoidal current assumption, the magnitude of the electric field distribution around the dipole can be determined as (noting that $\ell/\lambda = \lambda/2$):

$$E = \frac{60I}{r} \cdot \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \tag{4.17}$$

4.4.2 Directivity

The directivity of this antenna is clearly not much larger than that of the short dipole. The accurate value is:

$$\boxed{D\text{ (half wave dipole)} = 1.64} \tag{4.18}$$

this is equivalent to $\boxed{2.16\text{ dBi}}$ (relative to isotropic).

It is immediately clear that there is not a large difference between the gain of the half wave dipole and that of the short dipole. This initially does not make sense since a short dipole can be very much smaller than a dipole and hence cheaper and more practical. The primary reason for the popularity of the half wave dipole is its large and resonant input impedance—which was the problem with the short dipole.

Similarly, the directivity of a quarter wave monopole—which is the image theory equivalent of a half wave dipole—can be found as:

$$D\text{ (quarter wave monopole)} = 2.16 + 3 = 5.16\,\text{dBi} \tag{4.19}$$

The notation dBi is quite important and has been assumed until now. Very often antenna gain and directivity is quoted relative to a *half wave dipole* since this is a physically realizable antenna unlike the isotrope. The gain can thus be directly measured by comparing the signal strength received from a half wave dipole to that of the test antenna. When gain is quoted relative to a dipole it should be clearly stated and often this is done by using the notation dBd (decibels relative to dipole). The conversion between the two is evident:

$$\boxed{\text{dBi} = \text{dBd} + 2.16} \tag{4.20}$$

It is always important to ascertain which of these two references are used when gain is specified or quoted since many sources do not distinguish between the two—not an ignorable difference!

4.4.3 Input impedance

By analysis, the input impedance for thin half wave dipoles is:

$$\boxed{Z_{\text{in}} = 73 + j43 \quad \Omega} \tag{4.21}$$

This antenna is thus slightly longer than the length required for resonance. When a thin antenna is shortened by about 2% resonance can be obtained. As before, the quarter wave monopole has half the input impedance of the half wave dipole.

$$Z_{\text{in}}\text{ (quarter wave monopole)} = 36.5 + \text{j}21 \quad \Omega \tag{4.22}$$

The relatively large values of radiation resistance of these antennas makes for easy transfer of power and virtually lossless antennas when good conductors are used. Efficiencies are typically 99% or higher and losses can thus be neglected. This may be untrue in cases of extremely thin wires or high frequencies (>1000 MHz).

The impedance bandwidth of thin dipole antennas as defined by the VSWR 2 : 1 limitation is typically 5% of the centre frequency. For thicker antennas (small length to diameter ratios) this bandwidth can be larger.

4.5 The Folded Dipole

It is very seldom that folded dipoles of other values than half wave length (or slightly less to achieve resonance) are used. Folded dipoles are often used instead of normal dipoles for the following reasons:

- Mechanically easier to manipulate and more sturdy
- Larger bandwidth than normal dipoles
- Larger input resistance than a normal dipole
- Can offer a direct DC path to ground —for lightning protection.

The element centre opposite the feed can be "shorted" to the boom since this is a zero voltage point

This antenna's characteristics are again easily understood using the current moment technique. Clearly the currents in each arm are sinusoidally distributed as in a half wave dipole and are in the same direction. The current moment of this antenna is thus *double* that of the normal dipole. This implies:

$$R_{\text{in}} = 4(70) = 280 \quad \Omega \tag{4.23}$$

$$D \text{ (folded dipole)} = D \text{ (half wave dipole)} = 2.16\text{dBi} \tag{4.24}$$

$$E \text{ (folded dipole)} = 2E \text{ (half wave dipole)} \tag{4.25}$$

Folded dipoles typically have an impedance bandwidth (defined by the VSWR $2:1$ limit) of 10 to 12%. This increase relative to an half-wave

4.6 Exercises

Exercise 4.1: Radiation pattern of very short dipole

Purpose: *To illustrate the Radiation Pattern and input impedance of a very short (≈ Ideal) dipole and a half-wave dipole.*

1. From the SuperNEC Input Viewer, use `Add| Assembly| antennas|-sndipole` to pull up the dipole assembly dialogue box. Change the `End1` to `0 0 -0.05` and `End2` to `0 0 0.05` to obtain a very short dipole.
2. Using `Edit| Simulation Settings` click on `Radiation Patterns` and add a 3D pattern, 1 degree increment. Remember to click the `Add` button before closing the dialogue box. Simulate.
3. In the Output viewer, choose the `Radiation Patterns` tab, and plot it. To cut away a section choose `View| Exclude` and enter a phi cut from `-90 0` and a theta cut of `0 180` . Fig 4.8 shows the cut-away doughnut-shaped radiation pattern. (I find the most useful view of a 3D pattern is the Mesh type obtained via the `Type| Mesh` menu item.)
4. Click on the little button in the very bottom-left of the 3-D radiation pattern viewer, and the greyed-out UI controls will be brought to life, and a white line will appear around $\phi = 0$. Click on the button marked `2D` to get a 2D elevation cut (Note that you can vary the angle for the 2D cut request). Add a marker, and record your maximum gain in table 4.1. Do not close the 2D radiation pattern viewer.

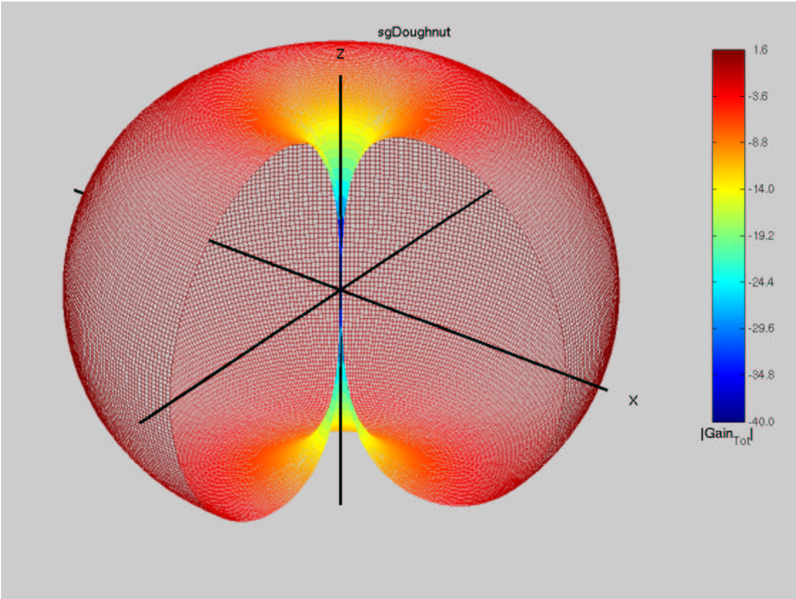

Figure 4.8: Doughnut-shaped radiation pattern of a very short dipole

5. From the Parameter vs Frequency, also record the input impedance of the very short dipole: a pretty highly-capacitive almost short-circuit!!
6. Delete all structures from the Input Viewer, and add a halfwave dipole using `Add| Assembly| antennas| sndipole`, accepting all the defaults.
7. Add a 2D radiation pattern in the xz plane by using `Edit| Simulation Settings` after deleting the 3D pattern that was used previously. Simulate, and overlay the radiation pattern on the 2D cut taken previously.

Table 4.1: Theoretical versus Simulated: Gain and Z_{in} for a short and half-wave dipole.

	Short Dipole		Half-wave Dipole	
	Z_{in}	Gain	Z_{in}	Gain
Theory				
SuperNEC				

Conclusion: There is very little difference between the radiation patterns of the very small dipole and the halfwave case; even the peak gains are very similar.

However, the input impedance of a very short dipole is almost a short circuit (and very capacitive at that), whereas the half-wave dipole is almost resonant and has a high enough resistive part to enable efficient power transfer.

It is therefore exceptionally difficult to deliver power into a small antenna, which is why a half-wave dipole is chosen: but not because it has a better gain than the small dipole!

Exercise 4.2: Varying the length of a dipole.

Purpose: *To consolidate why the half-wave dipole is the most useful for all practical purposes.*

1. Obtain a standard half-wave dipole by `Add| Assembly| antennas| sn-dipole` and accept the defaults.
2. Change the model frequency to 900MHz, remembering to Click the `Set` button to activate the re-segmentation of the dipole.
3. Using `Edit| Simulation Settings` add a frequency all the way from 10 to 900MHz (Enter `[10:900]` in the Frequecy entry.)
4. Add a single point Radiation Pattern at $\phi = 0; \theta = 90$ by clicking on the xy plane request, and change the phi entry from `[0 360 361]` to `[0 360 1]`. Plot the gains and obtain the figure 4.9. Note that in fig 4.9, the dipole is a half-wavelength long at its resonance at 300MHz; a full-wavelength long at its anti-resonance at 600MHz; 3 halfwavelengths at 900MHz, and a minute one-sixtieth of a wavelength long at 10MHz. Figure 4.10 shows the VSWR plot of the dipole across that frequency sweep.

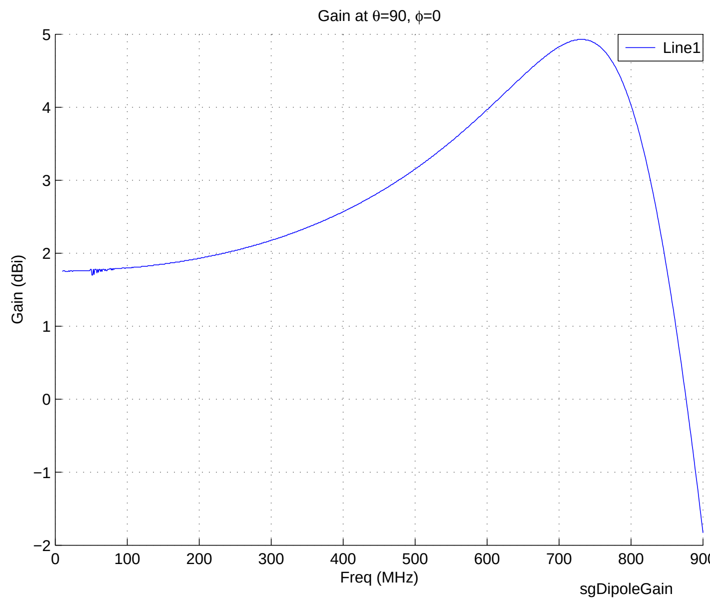

Figure 4.9: Gain variation of a Dipole with Frequency

5. It is instructive to view the impedance variation on the Smith Chart, as well as the real and imaginary part of the impedance. The real part of the impedance shows the anti-resonance very nicely. Record the peak gain obtainable from the Dipole, frequency at which it occurs, and the input impedance at that frequency.
6. Make sure that you `Exit` the Output Viewer (but not the input viewer!) in order to clear its notion of the available frequencies in the models it has collected. The `Edit| Simulation Settings` and remove all existing Radiation Patterns and select a 3D pattern at 1 degree increment.

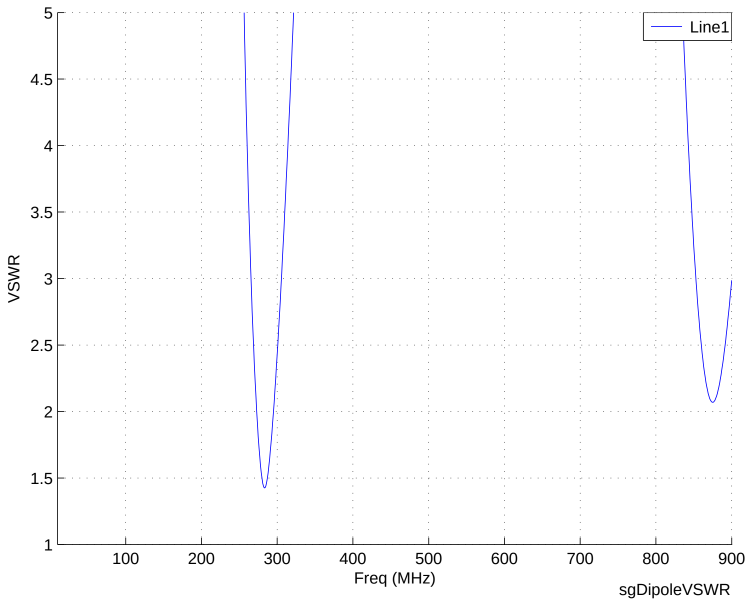

Figure 4.10: VSWR variation of a Dipole with Frequency

Table 4.2: Peak Gain from swept dipole

	Swept Dipole
Peak Gain	
Frequency	
Size in Wavelengths	
Input Impedance	

Change the frequency spec to `[300 600 (max gain freq) 900]`—just 4 frequencies, and simulate.

Plot the 3D patterns one at a time to see just what is happening to the pattern as the dipole gets longer (in terms of wavelength) at the higher frequencies.

Conclusion: Many "resonant" antennas (as opposed to "travelling-wave", or "slow-wave" antennas) are limited in their useful bandwidths by impedance not gain bandwidths. Longer antennas (in terms of wavelength) get more "lobey" in their patterns and push energy away from the intended directions into these lobes.

The increase in gain between 10MHz and 300MHz is minimal: and the difference in real-estate taken up by the antenna is vast: surely any cell phone designer would prefer a $1/60\lambda$ antenna spec than trying to find space for a $1/2\lambda$ antenna. But the impedance of the small antenna is just too difficult to feed efficiently.

Exercise 4.3: Monopole versus Dipole

Purpose: *To illustrate the differences between a dipole and a monopole.*

1. `Add| Assembly| antennas| snmonopole` will provide a standard quarter-wave vertical monopole at 300MHz.
2. Don't forget that a monopole has to be fed against a ground-plane! Use `Add| Ground` to specify a `Perfect` Ground type, with the currents Interpolated into the ground (the default for the Perfect Type).

   Depending on the status of your `View| Ground` Menu Item, you may now see a brown Ground Plane in the input viewer.
3. use `Edit| Simulation Settings` to add a Radiation Pattern in the yz plane, but change the Theta angles from `[0,360,361]` to `[-180,180,361]`, which comes to the same thing, but it keeps the radiation pattern viewer happier when a ground plane is present.
4. Simulate and record the `SuperNEC` simulated input impedance and peak gain in table 4.3

Table 4.3: Input Impedance and (grazing) Gain of a monopole

	Impedance (Z_{in})	Gain (at $\theta = 90°$)
Theory		
Perfect Ground		
0.25λ Ground		
0.5λ Ground		
1λ Ground		

5. Use texttttEdit— Remove Ground to remove the Perfectly conducting, infinitely large Ground Plane.

   Add a finite-sized ground plane using Add— Assembly— structures— snplate, changing the default size to 0.25m wide and 0.25m long. As you will see, this generates a wire grid in the xy plane under the monopole, of finite size.

   With the values given, you will note that the monopole is not on a wire junction on the `snplate`, hence no current will flow on it, giving rubbish results. You will note that three segments a side have been used, so select just the plate and click the `Edit` button, and change the `Number of Segments (Length)` from 0 (Auto segment) to 4. Do the same with the Width. You will now have a wire junction at the origin, and it will look like fig 4.11

   Simulate and record the results in table 4.3, and overlay the radiation pattern over the Perfect Ground case.
6. Select the `snplate` and specify a 0.5m by 0.5m plate, returning the `Number of Segments` entries to 0 to get it autosegmented.

   Again this has produced a plate with an odd number of segments (5) without a wire junction at the monopole. `Edit` the `snplate` again and

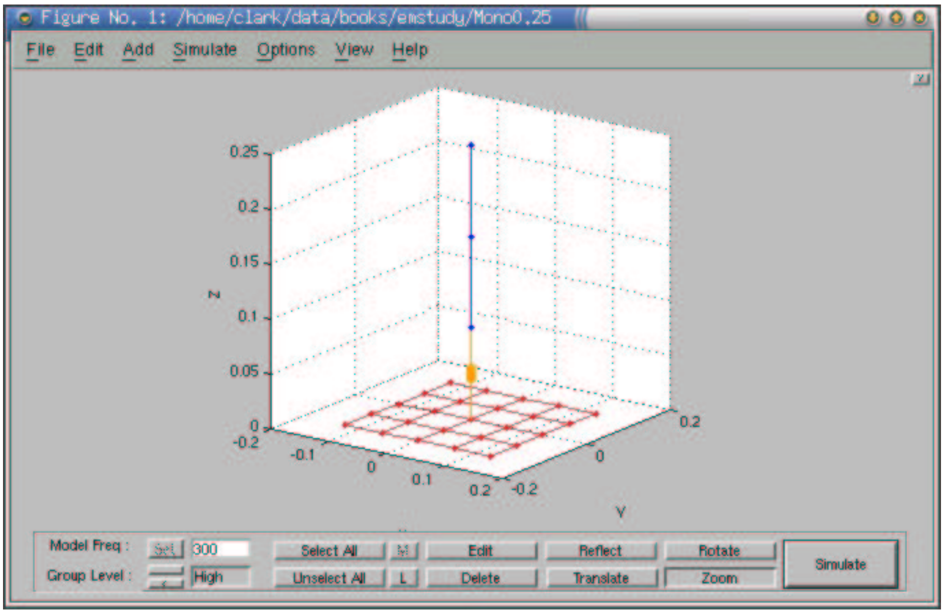

Figure 4.11: Monopole over a 0.25λ ground plane.

specify 6 segments in both the length and width `Number of Segments` entries.

7. Repeat for a 1m by 1m `snplate` (This time the autosegmentation does produce a valid junction with the monopole) The results are shown in fig 4.12.
8. If you have the full version of SuperNEC, repeat for a 5m by 5m plate. Although this is tricky to prove with zooming and repositioning, the autosegmentation does provide a valid junction with the monopole. You will need a coffee break for this one! The 5m by 5m uses 5103 segments as opposed to the 223 segments used by the 1m by 1m `snplate`.

Conclusion: As can be seen from fig 4.12 theory is all very marvellous if you neglect the obvious: No ground plane is infinitely large. Thus, no matter how big your ground plane is, it still looks tiny when seen edge on!!

Even the 5 wavelength groundplane did not make a difference to the grazing angle radiation of the monopole, and certainly is no where near the "5dB" gain quoted by every manufacturer I know of.

The very small ground plane (0.25 λ doesn't even pretend that it blocks radiation under the ground plane! This exercise shows the power of `SuperNEC` in simulating the real-world situation, not the theoretical one!

Exercise 4.4: Folded Dipole versus Dipole

Purpose: *To illustrate the properties of the Folded Dipole.*

1. `Add| Assembly| antennas| snfdipole` adds a Folded Dipole to the Input Viewer. In order to remain in the same frame of reference as the other

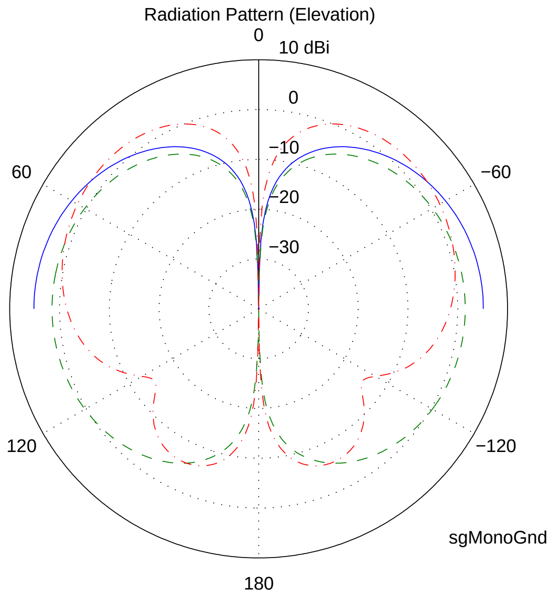

Figure 4.12: Monopole radiation patterns for a 0.25, 0.5 and 1 wavelength groundplane

basic antennas in this chapter rotate the default folded dipole by entering `[90 0 90]` in the `Orientation` entry in its dialogue box. This orientation will cause a vertical folded dipole. Again, experiment with the `View| Lock Aspect` to determine the most logical view of it.

2. Add a yz plane radiation pattern using `Edit| Simulation Settings`. The plot is shown in fig 4.13 which shows a significant skewing of the radiation pattern towards the element with the source segment. (Source is on the left of fig 4.13)

Conclusion: The rather large spacing and thick tubing used in the default Folded Dipole means that the assumption of similar current profiles on both vertical elements is incorrect. If you examine the SuperNEC output file (by issuing an appropriate `edit` command in the Matlab command window), you will notice that the current magnitudes on the source side are quite a lot higher than those on the other side of the Folded Dipole.

Experiment with thinner wires with closer spacing, and the effect will become less marked.

Note also in fig 4.13, that the severe nulls of the dipole in the upwards and downward direction have been softened somewhat, simply due to the presence of a radiating bit of metal in that direction (the end-pieces).

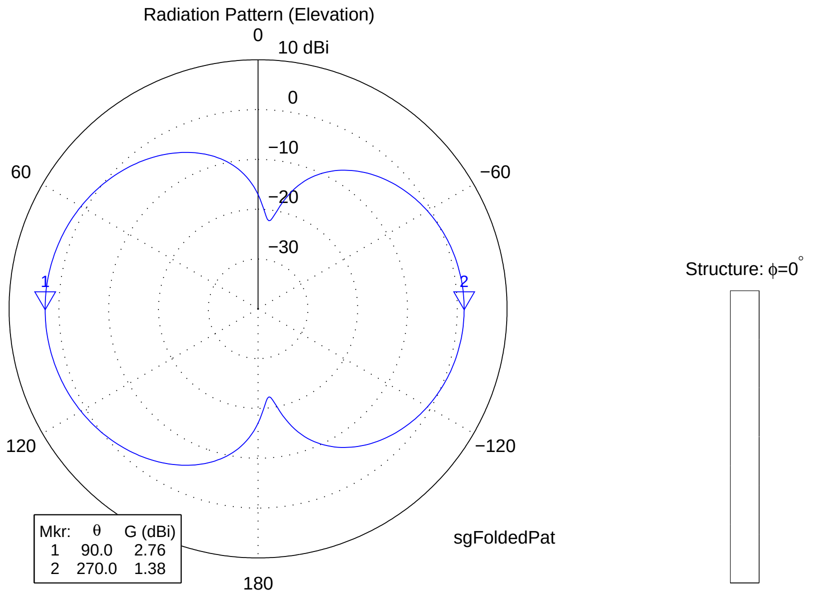

Figure 4.13: Radiation Pattern of a Folded Dipole, skewing towards the source.

4.7 Problems

4-1. Radiation Resistance Compare a plot of Radiation Resistance according to equation 4.13 with the input resistance obtained from SuperNEC for a range of ℓ/λ (Still short, though).

4-2. Radiation Resistance Compare a plot of Radiation Resistance according to equation 4.15 with SuperNEC.

4-3. Resonance It is stated that shortening a dipole's length by 2% it can be resonated. This amount actually changes with the thickness of the dipole. Run SuperNEC on a few dipoles of different thicknesses and iterate until resonance is achieved.

4-4. Resonance Plot the Real part of the impedance obtained from the previous problem against thickness of the dipole.

4-5. Short Dipole Using the techniques in the matching chapter attempt to create a stub match to the dipole in exercise 4.6. Compare the VSWR bandwidth obtainable by this method as compared to what you would be able to get on a half-wave dipole.

4-6. MonoPoles Most commercially available monopoles do not have extensive ground planes. As seen from Exercise 4.6, the input impedance already stabilises at the 0.25 wavelength groundplane, but the gain, especially at grazing angles, where is it most often required, is far worse than a dipole. (Most people motivate for monopoles because of the "extra 3dB"!!) Most monopoles are thus sold with three or four radial wires (only 0.25 λ long) that act as a ground plane. Construct such an antenna and simulate it.

4-7. Creating Assemblies Create an assembly which would easily allow the construction of a monopole with radial wires which emulate a ground plane. You must be able to specify Number of radials, thickness of radials and length of radials.

4-8. Creating Assemblies Modify the previous assembly to allow the radials to be tilted downwards by a given angle.

4-9. Using the Created Assemblies Determine the optimum downtilt angle to achieve resonance of a monopole with 4 radial wires as a groundplane.

Chapter 5

Array Theory

This chapter introduces array theory. An understanding of the fundamentals of array theory is necessary to understand the behaviour of more complex antennas, and to avoid the common pitfalls in arraying a collection of antennas.

5.1 Theory

MUCH OF ANTENNA THEORY consists of correctly adding field contributions at a point from all parts of an antenna, or antenna array. Note that *fields* must be used, not power, as proper vector addition of magnitude and phase must occur.

5.1.1 Isotropic arrays

Consider two isotropic sources, separated by d, having the same magnitude and phase.

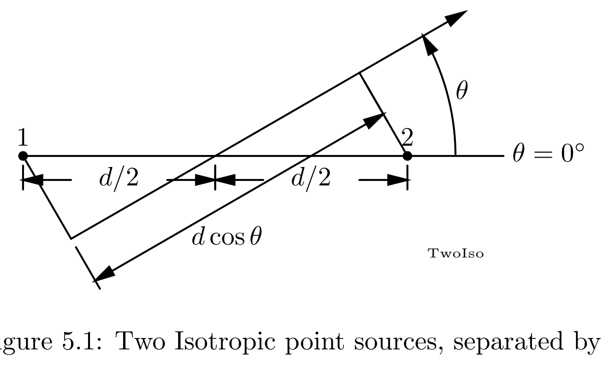

Figure 5.1: Two Isotropic point sources, separated by d

The far E-field is given by [Kraus and Fleisch, 1999, pg260]:

$$E = E_2 e^{j\psi/2} + E_1 e^{-j\psi/2} \tag{5.1}$$

where $\psi = \beta d \cos\theta = (2\pi d/\lambda)\cos\theta$ is the phase-angle difference between the fields from the two sources. If $E_1 = E_2 = E_0$, we get:

$$E = 2E_0 \cos(\psi/2) \tag{5.2}$$

For the special case of $d = \lambda/2$,

$$E = E_0 \cos\left(\frac{\pi}{2}\cos\theta\right) \tag{5.3}$$

which is shown in figure 5.2.

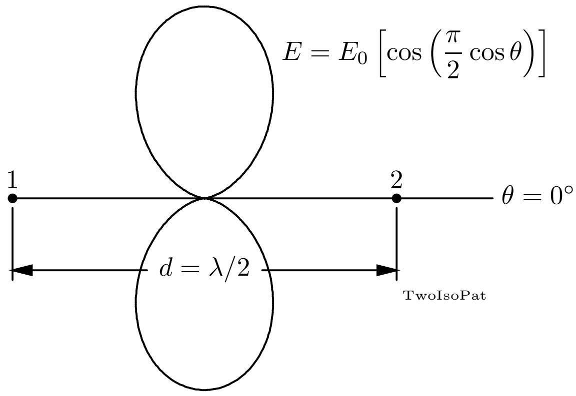

Figure 5.2: Two Isotropic Sources separated by $\lambda/2$

5.1.2 Pattern multiplication

The total field pattern of an array of *non-isotropic* sources is given by the multiplication of the *element* field pattern and the *array* field pattern.

Two short dipoles placed in echelon $\lambda/2$ apart. The *element* pattern is $k \sin\alpha$, a figure of eight perpendicular to the dipoles. From the above, the *array* factor is $\cos\left(\frac{\pi}{2}\cos\theta\right)$, a figure of eight parallel to the dipoles.

By pattern multiplication, we now get four (weak) lobes at 45° as seen in fig 5.3. Note that in this case, the overlap is very small. Ordinarily one wants the element and array pattern to have strength together.

5.1.3 Binomial arrays

If we take the two element isotropic array above, the (normalised) pattern is

$$E = \cos\left(\frac{\pi}{2}\cos\theta\right) \tag{5.4}$$

If we place another identical array one $\lambda/2$ away, we get a three element array with relative current magnitudes of 1:2:1.

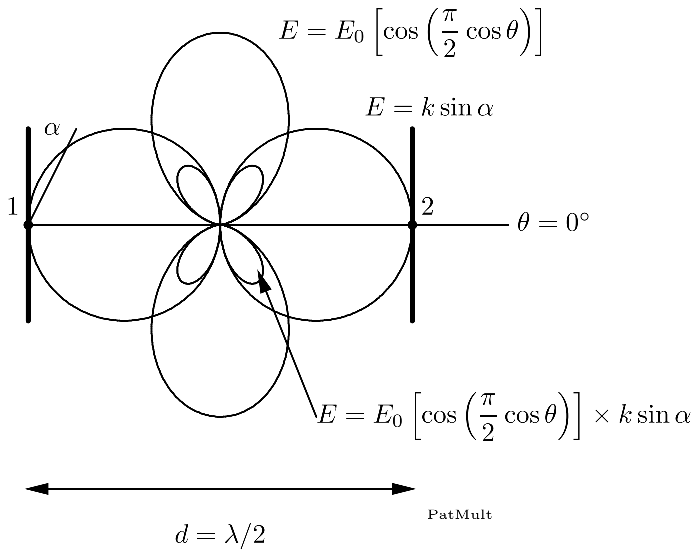

Figure 5.3: Pattern multiplication

By applying pattern multiplication, the pattern of this array is

$$E = \cos^2\left(\frac{\pi}{2}\cos\theta\right) \tag{5.5}$$

If this process is repeated, we will have a four source array with relative current magnitudes of 1:3:3:1. Clearly, continuing the process will provide source magnitudes given by Pascal's triangle.

$$\begin{array}{c} 1 \\ 1\ 1 \\ 1\ 2\ 1 \\ 1\ 3\ 3\ 1 \\ 1\ 4\ 6\ 4\ 1 \\ 1\ 5\ 10\ 10\ 5\ 1 \end{array}$$

Clearly, the pattern multiplies each time, and we get that the pattern of an array of n sources is:

$$\boxed{E = \cos^{n-1}\left(\frac{\pi}{2}\cos\theta\right)} \tag{5.6}$$

This array has *no minor lobes*, but its directivity is less than that of an array of the same size with equal amplitude sources.

Generalising, what has been applied here is an *amplitude taper* whereby the outer elements of an array receive less current, as this improves sidelobe levels. It can be further generalised to show that the sidelobe pattern is given as a function of the Fourier Transform of the Amplitude taper in an analogous way to Window Functions in Digital Signal Processing. If the Window function has sharp transitions eg a Rectangular, or Uniform array, the Sidelobe pattern is a Since function, with high sidelobe levels.

In general, it is difficult to achieve these power ratio's at each element of the array using power splitters, and it is far more common to feed each element with equal power—which is a uniform array.

5.1.4 Uniform arrays

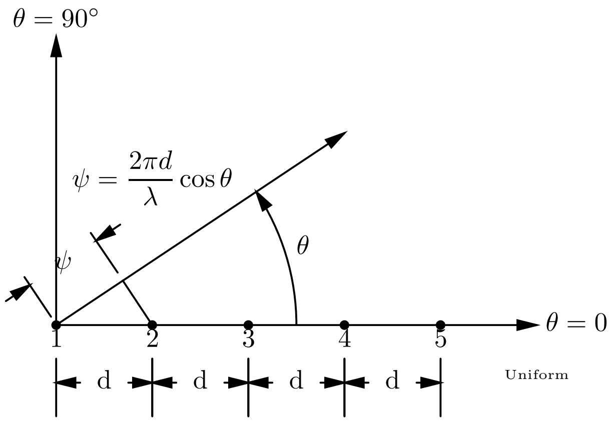

Figure 5.4: Uniform linear array of isotropic sources.

If instead, we have an array of equal amplitude, E_0, and spacing, d, (not necessarily equal *phase* as shown in figure 5.4, the far field E-field at angle θ is given by:

$$E = E_0\left(1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \cdots + e^{j(n-1)\psi}\right) \tag{5.7}$$

where $\psi = \beta d\cos\theta + \delta$, δ being the progressive phase difference between the sources. (Phase reference is source 1)

Multiplying (5.7) by $e^{j\psi}$ and subtracting (5.7) from the result yields: (Since (5.7) is an infinite series, we adopt the usual geometric series method.)

$$\begin{aligned}(1 - e^{j\psi}) &= E_0(1 - e^{jn\psi}) \qquad \text{ie:}\\ E &= E_0\left(\frac{1 - e^{jn\psi}}{1 - e^{j\psi}}\right)\end{aligned} \tag{5.8}$$

This can be manipulated using half-angle expansion, and assuming a new phase reference in the middle of the array, we get:

$$\boxed{E = \frac{\sin(n\psi/2)}{\sin(\psi/2)}} \tag{5.9}$$

As $\psi \to 0$, $E = nE_0$, ie the E field of n sources at the same point, as it should! This is the maximum E field attainable. Two special cases of maximum field are of interest—*broadside* and *end-fire* arrays.

Broadside (as in a naval galleon :-) fires its maximum at $\theta = 90°$. For max field, $\psi = 0 = \beta d\cos 90° + \delta$, hence for max broadside field,

$$\boxed{\delta = 0} \tag{5.10}$$

This means that there is *no progressive phase shift*, ie that all sources are fed *in-phase*.

Endfire has its maximum at $\theta = 0°$, hence $\psi = 0 = \beta d \cos 0° + \delta$, hence for max endfire field,

$$\delta = -\beta d = -\frac{2\pi}{\lambda} d \tag{5.11}$$

As an example, if the sources a spaced a quarter wavelength apart,

$$\delta = -\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = -\frac{\pi}{2} = -90° \tag{5.12}$$

ie that there needs to be a 90° progressive phase shift between sources (equalling the "quarter wavelength apart" spatial phase.

Beamwidth

From (5.9) it can be seen that the nulls occur when $\sin(n\psi/2) = 0$ (with the proviso that $\sin(\psi/2)$ cannot also be zero!)

We are interested only in the *first null*, and this occurs at: $n\psi/2 = \pm\pi$, or $\psi = \pm\frac{2\pi}{n} \quad (= \beta d \cos\theta_0 + \delta)$, where θ_0 is the angle of the first null.

Hence the first null occurs at

$$\theta_0 = \cos^{-1}\left[\left(\pm\frac{2\pi}{n} - \delta\right)\frac{\lambda}{2\pi d}\right] \tag{5.13}$$

For the *Broadside case*, we are interested in the beamwidth at $\theta = 90°$, hence we use the complementary angle $\gamma = 90 - \theta$. Recall that for broadside, $\delta = 0$, so that the first broadside null is given by:

$$\boxed{\gamma_0 = \sin^{-1}\left(\pm\frac{\lambda}{nd}\right) \qquad \text{Broadside}} \tag{5.14}$$

If the array is large, $nd \gg \lambda$ and the argument to the arcsine is small, (for small angles $\theta \approx \sin\theta$):

$$\gamma_0 = \frac{1}{nd/\lambda} = \frac{1}{L/\lambda} \tag{5.15}$$

where L is the length of the array $L = (n-1)d \approx nd$ for a large array.

The BWFN is obviously twice this angle, hence:

$$\text{BWFN} = 2\gamma_0 \approx \frac{2}{L/\lambda} \quad [\text{rad}] = \frac{114.6°}{L/\lambda} \tag{5.16}$$

For most purposes, we can say that HPBW≈BWFN/2, hence

$$\boxed{\text{HPBW} = \frac{57.3°}{L/\lambda}} \tag{5.17}$$

For the *Endfire case*, recall that $\delta = -\frac{2\pi}{\lambda}d$, hence the first null occurs at

$$\boxed{\theta_0 = \cos^{-1}\left[\pm\frac{\lambda}{nd} + 1\right] \qquad \text{Endfire}} \tag{5.18}$$

Recognising that we wish to use the small angle approximation again, ($\theta \approx \sin\theta$) we convert to a sin via $\cos 2\alpha = 1 - 2\sin^2\alpha$

$$1 - 2\sin^2\left(\frac{\theta_0}{2}\right) = \pm\frac{\lambda}{nd} + 1 \tag{5.19}$$

Hence

$$\sin\left(\frac{\theta_0}{2}\right) = \sqrt{\mp\frac{\lambda}{2nd}} \approx \theta_0/2 \tag{5.20}$$

As before, using $L \approx nd$, the first null angle is:

$$\theta_0 = \sqrt{\frac{2}{L/\lambda}} \tag{5.21}$$

$$\text{BWFN} = 2\theta_0 = 2\sqrt{\frac{2}{L/\lambda}} \quad [\text{rad}] = 114.6^\circ\sqrt{\frac{2}{L/\lambda}} \tag{5.22}$$

and hence

$$\boxed{\text{HPBW} = 57.3^\circ\sqrt{\frac{2}{L/\lambda}}} \tag{5.23}$$

5.2 Exercises

Remember that array theory assumes isotropic sources, which do not physically exist. Obviously, SuperNEC cannot simulate them! A dipole, however, radiates equally well in all directions in its azimuth plane, so they can be used to illustrate array theory in azimuth—with the proviso that you remember that a half-wave dipole (element) gain is 2.16dBi, not 0dBi. Short dipoles (0.1λ) produce 1.76dBi.

Exercise 5.1: Pattern Multiplication

Purpose: *To illustrate Pattern Multiplication in Array Theory. ie that the pattern due to the array, and the pattern due to the element is multiplied to form the final pattern of those elements in that array.*

1. Pull up the `sgarray` assembly as shown in fig 5.5

   The `sgarray` assembly can create a 2-dimensional array of vertical dipoles, with progressive phase shifts in the vertical and horizontal directions.

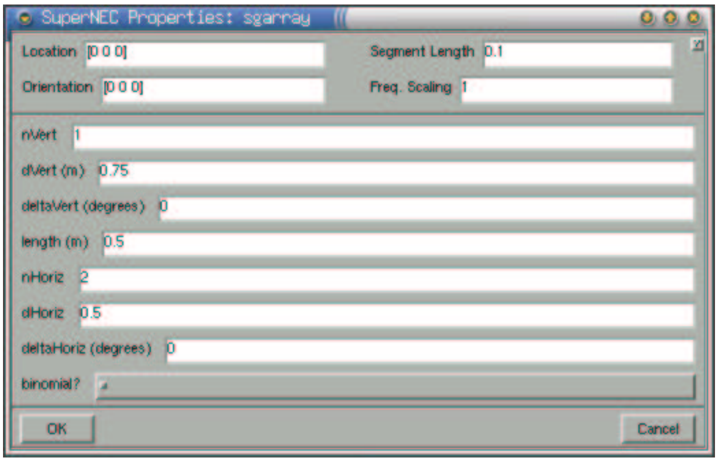

Figure 5.5: Dialogue box of the `sgarray` assembly

The default configuration creates a 2 element horizontal array, spaced $\lambda/2$ apart at the default frequency of 300MHz, fed in-phase. Accept the defaults. Use `Edit| Simulation Settings` to add an azimuth radiation pattern (in the xy plane). After simulation, plot the radiation pattern; *do not close the radiation pattern viewer.*

Note that there is no radiation in the x-axis: although the dipoles are fed in-phase, the *spatial* phasing is exactly $\lambda/2$ out-of-phase, causing absolute cancellation in that direction. This pattern is due to the *array*: remember that vertical dipoles are isotropic in azimuth!

2. Now go back to the input interface, Select All, and Delete. Add an `sndipole` assembly, but rotate it to make it horizontal: change the default orientation in the `sndipole` dialogue to `[90,0,0]`. Simulate and overlay the radiation pattern plot on the existing one. (Note that deleting the structure did not delete the radiation pattern specification)

   A single horizontal dipole has a pattern with maximum radiation in the x-axis and minimum radiation in the y-axis. This is the *element* pattern.
3. What would happen if we used this element in the array we had earlier? Without closing the radiation pattern viewer, go back to the input interface, Select All and delete. Then add a default `sgarray` except that the orientation must be changed to `[90,0,0]`.

   (Note that the orientation change by 90° gets internally converted to radians, and since you should be viewing without "Lock Aspect" on, the dipoles shown in the input interface will be at an angle other than 90. Note that the axis scale is 10^{-17} though!)

   Overlay the radiation pattern, and you should have something that looks like fig 5.6

Conclusion: This exercise demonstrates that an array has a radiation pattern purely due to the *physical* spacing, or *spatial phasing* of the radiating elements (the array pattern, or factor). The pattern due to each element in

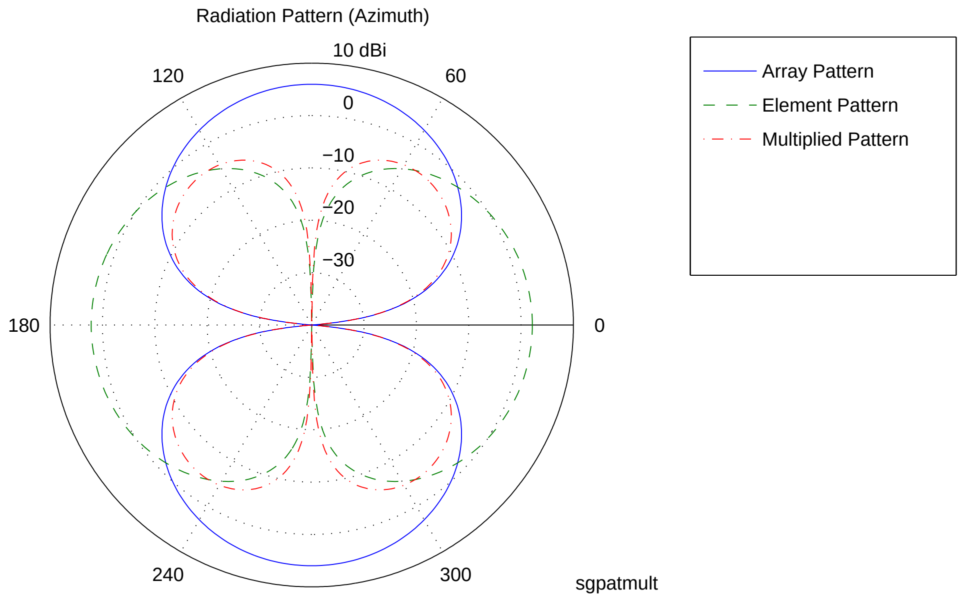

Figure 5.6: Demonstration of Pattern Multiplication.

the array is multiplied by the array pattern to produce the final radiation pattern due to those elements in that particular array.

If one is silly enough to put an element which radiates well in the direction that the array does not, (and vice versa) then one gets the rather pathetic radiation pattern shown in fig 5.6.

Obviously, the ideal occurs when *both* the element and the array have a pattern in the *same* direction, but this (silly) example demonstrates pattern multiplication very nicely.

Exercise 5.2: Broadside and Endfire

Purpose: *To illustrate Broadside and Endfire from the same array, by simply changing the* feeding *phasing, the* spatial *phasing being the same.*

1. Pull up the `sgarray` dialogue and specify 16 dipoles, horizontally apart by $\lambda/4$ (0.25m at 300MHz), fed in-phase ($\delta = 0$) as shown in fig 5.7 *NB: not the default spacing of 0.5m ($\lambda/2$ at 300MHz).*

2. Specify an xy-plane radiation pattern, and plot it. Do not close the plotting window.

3. Going back to the input editor, click the `Select All` and the `Edit` button. Make the progressive phase shift "`deltaHoriz`" −90 degrees.

4. Plot the new pattern as an overlay to the previous one, and you should get something like fig 5.8

   Note that the broadside gain is larger than the endfire gain: it is the "Robbing Peter to pay Paul"principle—remember that gain (in one direction)

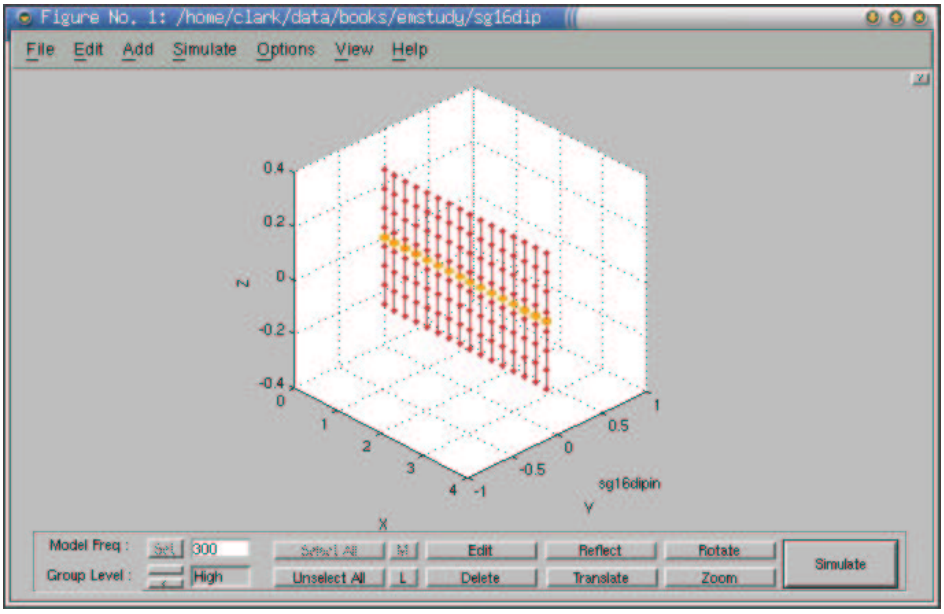

Figure 5.7: Uniform horizontal array of 16 dipoles.

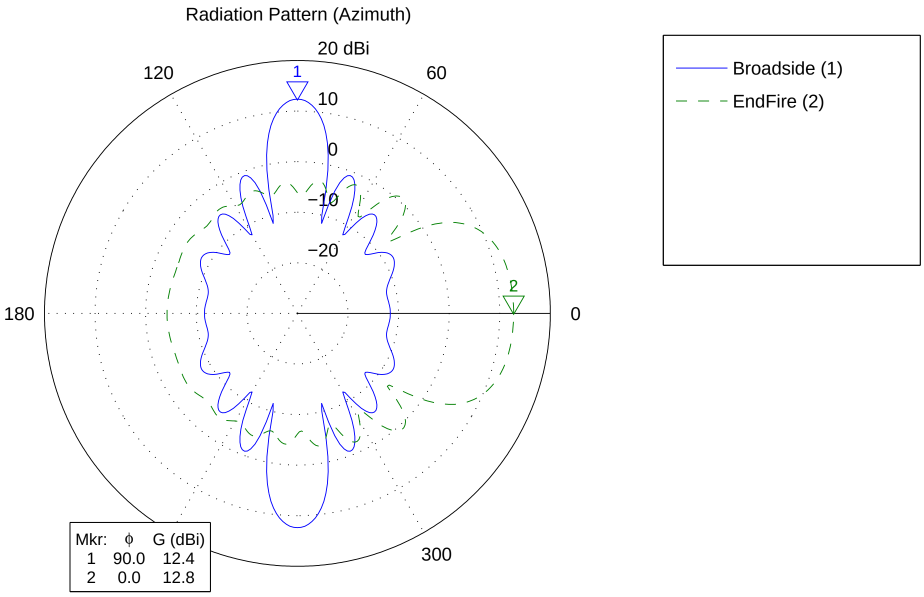

Figure 5.8: A 16-dipole array radiation pattern in Broadside and Endfire configuration

is *only* achieved at the expense of gain in another direction; and that to obtain gain in a direction, array *length* must be present. If you look at the array in the broadside sense, you see a lot of it: hence higher gain. If you look at it in the endfire sense, all you see is a dipole, (hiding all the other dipoles behind it—the array does not have much “length”: hence a lower gain, with a more “rounded” shape.

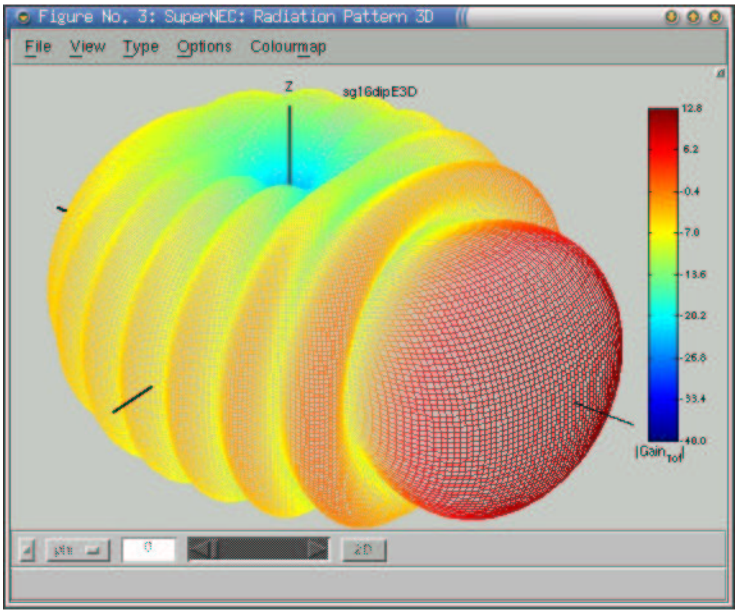

Figure 5.9: 3-D radiation pattern of the 16 dipole array in end-fire configuration.

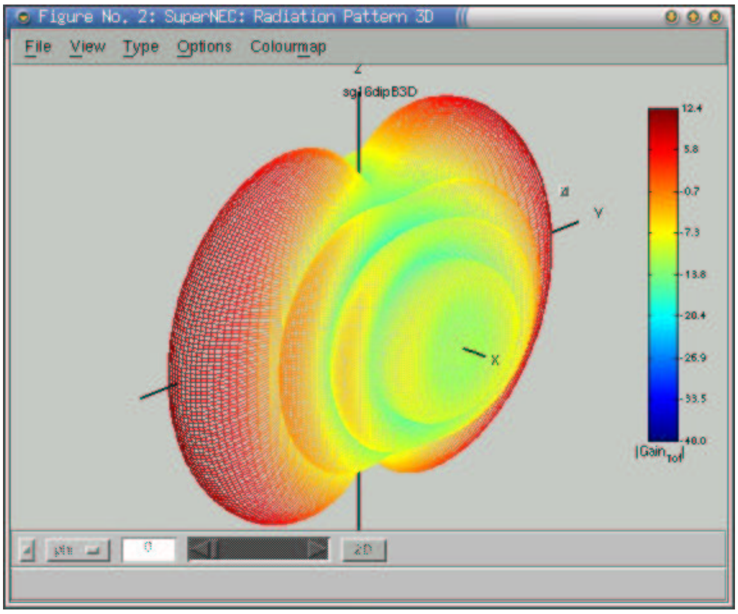

Figure 5.10: 3-D radiation pattern of the 16 dipole array in broadside configuration.

Fig 5.9 shows a three-dimensional radiation pattern of the 16-dipole array in end-fire configuration, and fig 5.10 shows the broadside configuration.

Note that an array needs *length* in the dimension that it is attempting to compress.

Note that the Endfire case has a property whereby, for $\lambda/4$ spacing $-90°$ it produces a beam towards the right. If the phasing were changed to $+90°$, it fires towards the left as shown in fig 5.11

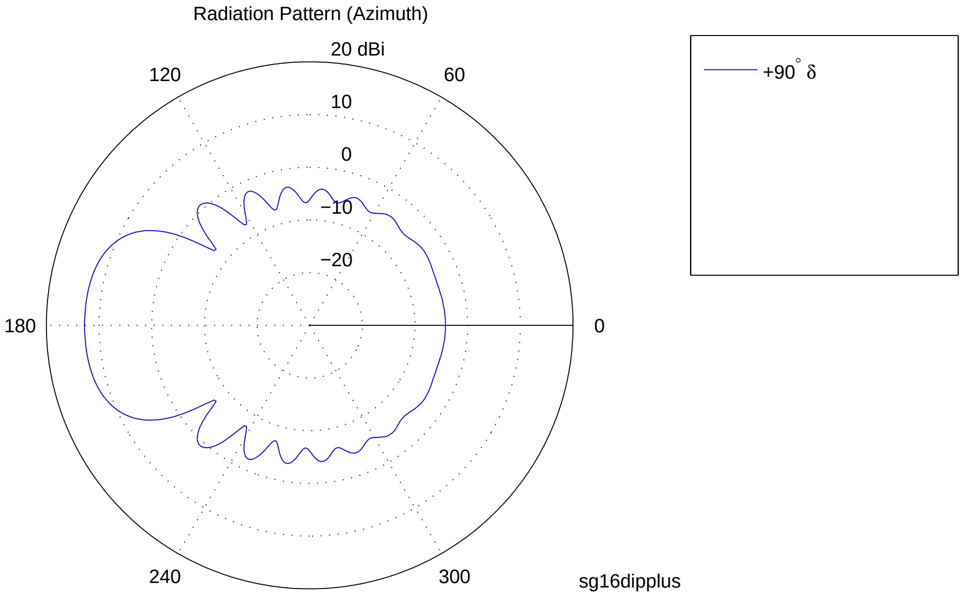

Figure 5.11: EndFire array of 16 dipoles with positive progressive phase shift of 90°

This leads to an interesting mathematical conundrum: What is the difference between $+180°$ and $-180°$? This occurs (if you solve the EndFire case equation at a spacing `dHoriz` of $\lambda/2$. In this special case, the endfire case fires both ways, as shown in fig 5.12.

Conclusion: An array can change its radiation pattern direction by changing the progressive phase by which it is *fed*. In this example, feeding all elements *in-phase* produces a broadside radiation pattern; feeding them at a progressive phase of $+90°$ or $-90°$ produces an endfire radiation pattern.

Note that in the endfire case, the *feeding* phase exactly equals the *spatial* phase. ie The radiation from the first dipole (fed at $0°$) reaches the second dipole, spaced $\lambda/4 = 90°$ apart, exactly $90°$ out-of-phase: thus the second dipole must be *fed* $90°$ in order to constructively interfere with the first dipole's radiation.

This can be generalised for the endfire case: the *feeding* phase must equal the *spacing* phase to boost the endfire condition. The generalisation becomes more difficult when you are not in total control of the feeding phase—which is the case in the parasitic elements of a Yagi-Uda antenna.

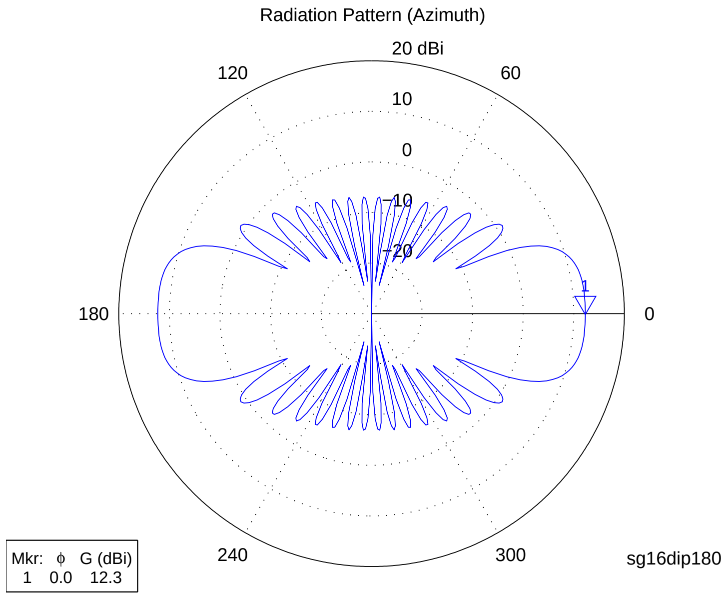

Figure 5.12: Special case, where “endfire” has to mean *both* ends!

Exercise 5.3: “Electrical” Downtilt

Purpose: *To illustrate “electrical” downtilting of a typical cellular vertical “8 stack” by feeding each element with a progressive phase shift.*

In cellular systems (GSM etc), base station antennas are typically vertically stacked dipoles—to compress the pattern in the elevation plane, whilst still being omnidirectional in the azimuth plane (for a non-sectored cell).

Earlier systems used mechanical downtilting to limit the coverage of a particular cell (it may sound odd, but that is ultimately the goal in a dense cellular environment), but the downtilting also causes an “uptilting” of the opposite backlobe (even in a sectored panel). “Electrical” downtilting brings down the entire pattern, helping to limit the cell’s coverage, and thereby promoting easier frequency re-use.

1. Pull up the `sgarray` dialogue box, and request 8 dipoles vertically (`nVert`), and change the default horizontal number from 2 to 1 (`nHoriz`). The resulting dialogue box is shown in fig 5.13.
2. using `Edit| Simulation Settings` add a radiation pattern in the yz-plane, simulate, and plot the radiation pattern, asking for the Structure to be displayed.
3. From array theory, (verify the answers), we get:

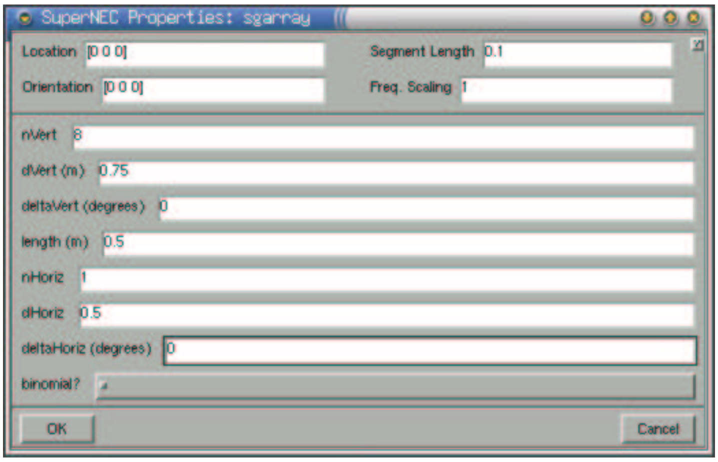

Figure 5.13: Dialogue box for a vertical eight-stack cellular dipole array.

Downtilt	Fed Phase
$0°$	$0°$ (broadside)
$5°$	$-24°$
$10°$	$-47°$
$20°$	$-92.3°$
$30°$	$-133°$

The "Fed Phase" column in the table refers to the amount of additional phase we need to add to each successive dipole. (The way the theory is defined, we start at the top, hence the negative phasing. You could easily start at the bottom with positive values of the same magnitude, but `sgarray` follows the usual left-most-is-phase-reference theory: in a vertical array, left is top!)

4. In the same way as before, leave the radiation pattern viewer on the screen, go back to the input editor, click on `Select All`, then `Edit` and successively enter the downtilt requirement to be 0, 10, 20 , 30 , 40 degrees, ie the progressive phasing (`deltaVert`) to be 0, $-24, -47, -92, -135, -174$ degrees. After each simulation, click the `Overlay` flag, and plot over the pre-existing plot. You Should obtain something like fig 5.14

   Fig 5.14 can be difficult to decipher, but demonstrates much: I have made all linetypes solid, for easier viewing, if you know what you are looking for! The first pattern, (in blue, for those with a colour version of this document) is at $0°$, or broadside—since theta starts at the zenith angle, marker 1, displays $270°$ for this at a gain of 10.8dBi.

   Next is a downtilted pattern at $10°$ (hence marker 2 shows $260°$ at 10.8 dBi.

   Next at $20°$ (250) at 10.6dBi (marker 3); $30°$ (240) at 9.52dBi (marker 4 and 6); $40°$ (230) at 8.06dBi (marker 5 and 7);

   Note that marker 6 and 7 denote increasing "uptilt" sidelobes associated with the 30 and 40 degree "downtilts" respectively.

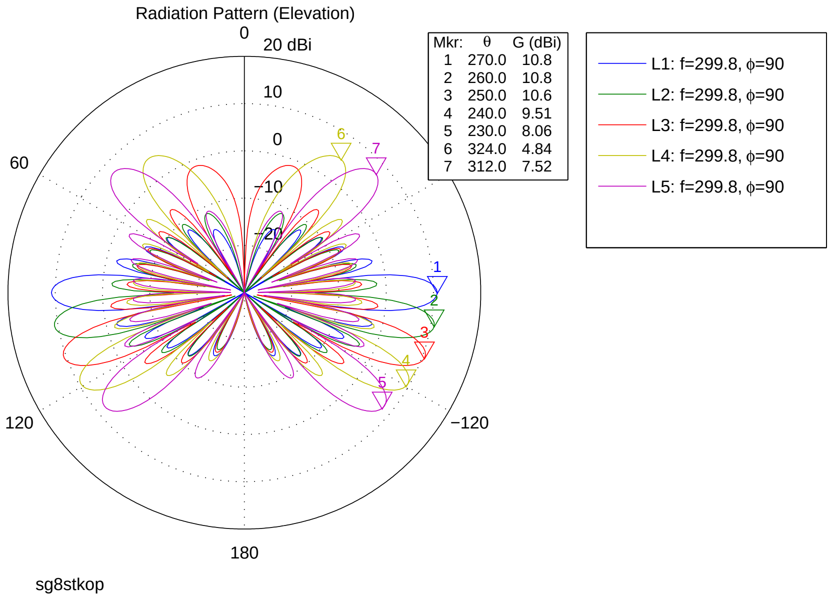

Figure 5.14: Different feeding phases for a vertical eight-stack dipole array, causing electrical downtilt.

Downtilt (°)	Gain (dBi)	Marker
0	10.8	1
10	10.8	2
20	10.6	3
30	9.52	4
40	8.06	5

Conclusion: It is possible to achieve downtilting of a vertical array by changing the phase progression to each element of the array. In this way, the uptilting of the backlobe is avoided, which means less interference with adjacent cell clusters in a cellular system.

Note that the greater the degree of downtilt is required, the less gain is achieved. In fig 5.14 the degradation in peak gain is clear, as shown in table 3

Also note that the sidelobes begin to perk up when the demanded radiation angle becomes large: for a 30° downtilt, marker 6 shows 4.84dBi and for a 40° downtilt a 7.52dBi uplobe, vs. a 8.06dBi downlobe.

Clearly, then, array theory is wonderful, but don't push the limits!!!

Exercise 5.4: Interferometer

Purpose: *To illustrate constructive and destructive interference when the sources are far apart.*

Interferometer's are widely used in Radio Astronomy because although they produce many sidelobes, the central beam is incredibly narrow, allowing for greater *resolution* to be obtained from the telescope—crucial to resolve one star from its close brother. The disadvantage of also "seeing" other stars in its many sidelobes is handled by lots of Digital Signal Processing.

One also sees unintentional interferometers: there exists a belief that one can illuminate a large area by using two antennas spaced far away from each other (at the same frequency). However, insomuch as a receiver "sees" both transmitters, it receives the interfered pattern—not the desired result!

1. To illustrate the pattern from the *array*, we use two dipoles operating as if they were isotropic, ie we use them in their azimuth pattern. Pull up the `sgarray`, and specify the horizontal distance between the two default dipoles (`dHoriz`) to be 10m (=10 wavelengths at the default frequency of 300MHz).
2. Using `Edit| Simulation Settings`, add a radiation pattern in the xy-plane, and simulate; the pattern will be as shown in fig 5.15

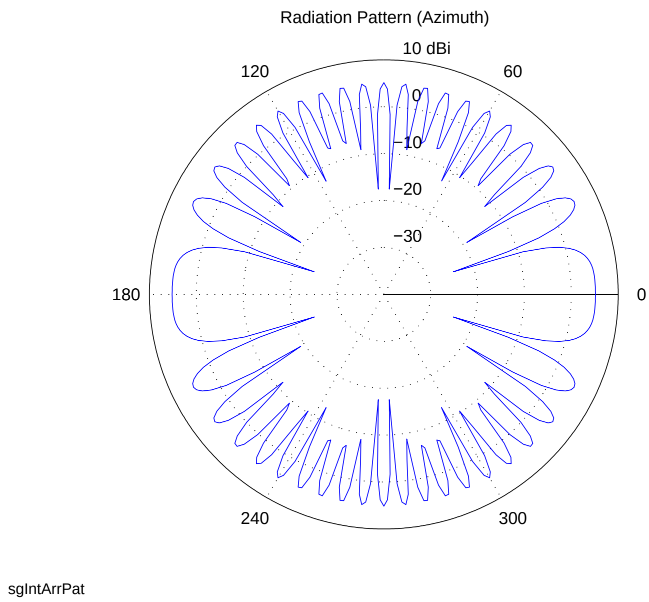

Figure 5.15: Radiation Pattern of a 10λ interferometer due to the *Array*

The radiation pattern shown in fig 5.15 is the pattern of any two isotropic sources placed 10 wavelengths apart. This array pattern will be modified by the element pattern of whatever element you place in the array. Nevertheless, any two antennas, fed in-phase and at the same frequency will have such a pattern due to constructive and destructive interference.

If your goal was to achieve omnidirectional coverage using two antennas spaced far apart, you have missed your goal!

3. A standard interferometer uses a horizontal dipole above the ground as its element in the array. If you recall, a horizontal dipole has a maximum

radiation upwards and no radiation at grazing angles. If we place these elements into the Interferometer array, we get pattern multiplication, so that the broad lobes at 0 and 180 degrees in fig 5.15 are cancelled.

Build up the interferometer in several steps:

(a) In the input editor click `Select All` followed by `Delete` to get a clean slate.
(b) Using the `sndipole` assembly (`Add| Assembly| antennas| sndipole`) modifying the Orientation vector to `[90,0,90]`.
(c) Use the `Translate` button of the input interface and enter the translation vector as `[10,0,0]` and make sure you change `Move` to Duplicate (once only). Depending on whether `View| Lock Aspect` is on, the display may look a bit odd with one or more axes displayed with a 10^{-17} scale. Locking the Aspect view gives a truer reflection!
(d) Click the `Select All` button on the input interface and use the `Translate` button again to move the whole array up by three-quarters of a wavelength: `[0,0,0.75]`, using the `Move` option.
(e) Add a ground plane with `Add| Ground`. Choose a perfect ground.
(f) Add a radiation pattern in the xz-plane using `Edit| Simulation Settings`, but modify the Theta vector from `[0,360,361]` to `[-180, 180, 361]` (First delete any old pattern specification which you may still have in the Simulation Settings.)

4. Simulate and plot the radiation pattern—it should look similar to fig 5.16

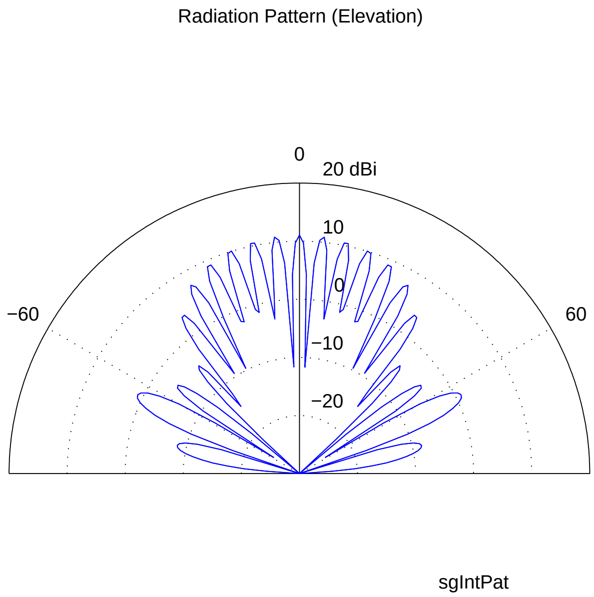

Figure 5.16: Radiation Pattern of an Interferometer consisting of two horizontal dipoles 10λ apart, 0.75λ above a perfect ground.

Conclusion: Any array which has elements far apart will have a pattern with many lobes: sometimes desirable, sometimes not. For a proper interferometer, the requirement is for a high gain, but very narrow main lobe.

Exercise 5.5: Binomial Array

Purpose: *To illustrate the difference between a Uniform and a Binomial Array.*

A Binomial Array is a special case of applying an *Amplitude Taper* to an array, whereby the outer elements receive less of the transmitting power than the central ones, by means of a specialised power splitter. A Uniform Array, on the other hand has a uniform amplitude, which then abruptly changes to zero at the edges of the array. A normalised comparison of the amplitude taper is shown in fig 5.17.

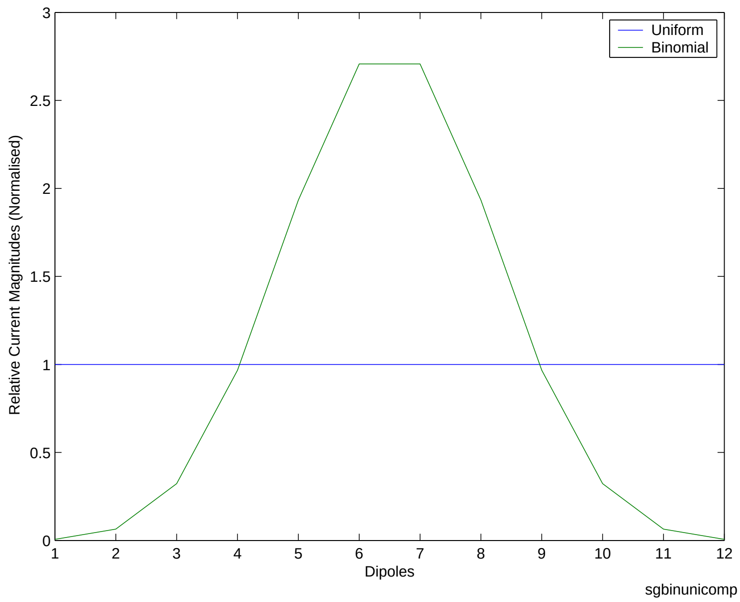

Figure 5.17: A comparison of the amplitude tapers applied to the uniform and binomial arrays, (12 dipoles)

As seen in fig 5.17, the uniform taper comes to an abrupt end at the edges of the array causing significant sidelobes, whereas the array is truncated much more gently by the binomial taper, which has most of its current in the middle of the array.

1. As usual, pull up the `sgarray` dialogue box and specify 12 horizontally spaced dipoles, (`nHoriz`), and use the rest of the defaults.
2. Specify an xy plane radiation pattern, and plot.
3. Leaving the radiation plotter on the screen, go back to the input editor screen and click `Select All` followed by `Edit`. Simply click the `binomial?` checkbox, and simulate again.
4. Overlay the radiation pattern on the original plot, and you will have something that looks like fig 5.18

The difference is more easily seen in Rectangular plot form, (using the `View| Rectangular` menu, and restricting the plot to 180° by `Options| Limits|`

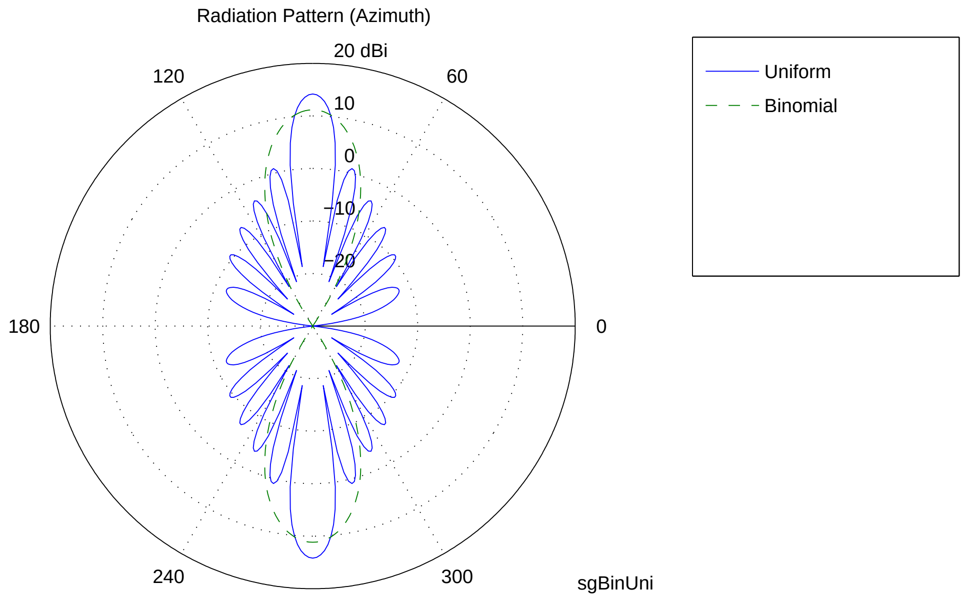

Figure 5.18: Comparison between the radiation patterns of a 12-dipole Uniform and Binomial array in Broadside configuration.

`Angle`, we get fig 5.19.

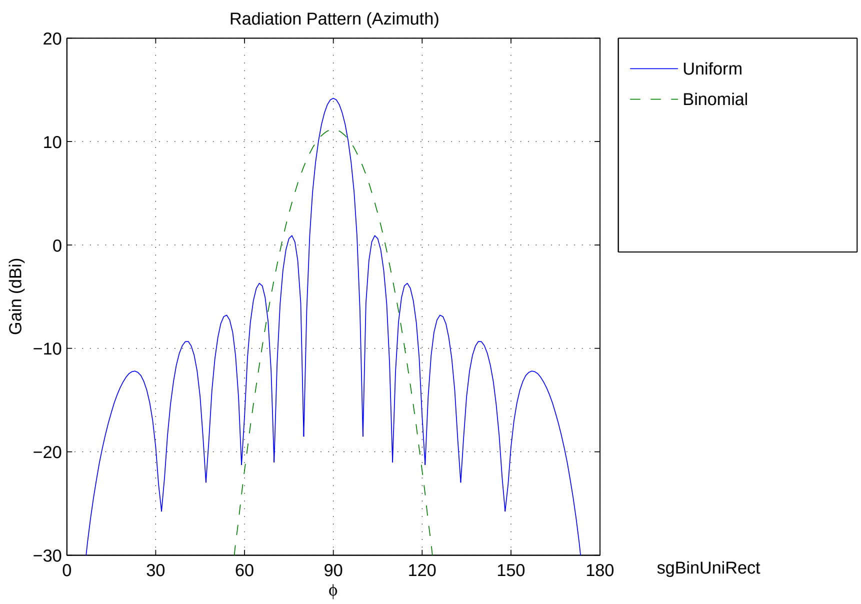

Figure 5.19: Rectangular view of the comparison between a binomial and uniform linear array pattern

Conclusion: As shown in fig 5.18 the binomial pattern is beautifully smooth, with a complete absence of sidelobes, but the uniform array has sidelobes only about 13 dB down on the main lobe. The uniform main lobe is slightly higher than the binomial version. You will notice that the first sidelobe of a Sinc function $((\sin(x))/x)$ is about 13dB below the main lobe. Hence the extensive use of using the inverse Fourier Transform of the desired pattern to define the required Amplitude Taper!

Exercise 5.6: Large Square Array

Purpose: *To illustrate the fact that the array itself has to have* length *in the dimension that it is attempting to "squash" the gain out of. (Remember: Peter only gets gain by robbing Paul) Hence the need for the "Square Kilometer Array (SKA)" radiotelescope.*

1. Pull up `sgarray`, and request a 10 by 10 array of short dipoles at equal (square) spacings. ie Change: `nVert` to 10; `dVert(m)` to 0.5; `length(m)` to 0.1; `nHoriz` to 10.

   The input editor should show an array similar to fig fig:sgSKAin. Note that the dipoles had to be shortened so as to prevent full-sized dipoles intersecting on the square grid!

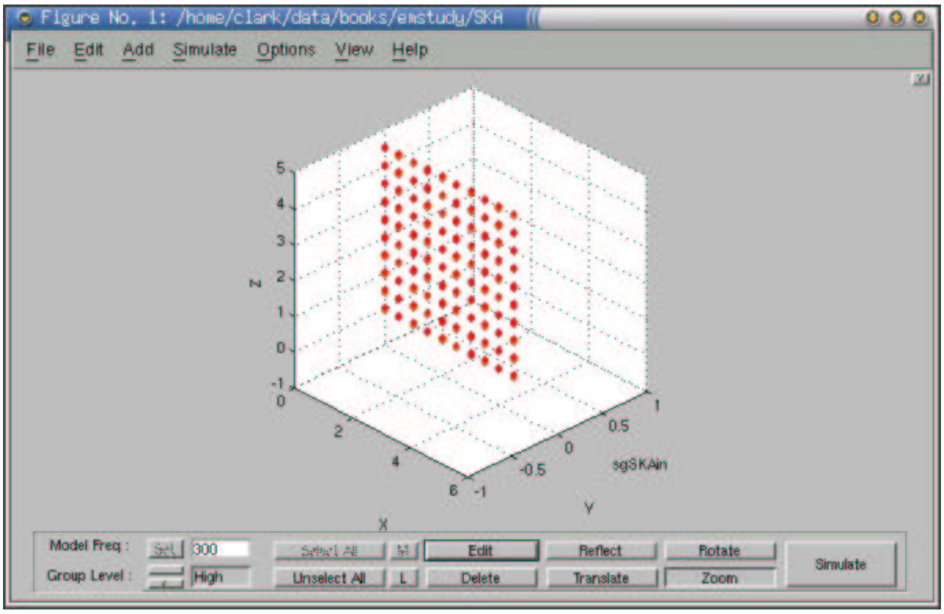

Figure 5.20: 10 by 10 array of short dipoles.

2. Edit the Simulation Settings to include a 3D pattern, and simulate. Depending on the speed of your computer, you may need a coffee break about now :-) The 3D pattern of the uniform square array is quite spectacular, with sidelobe after sidelobe as shown in fig 5.21
3. Going back to the input editor click `Select All` and `Edit` as usual and check the `binomial?` check box. This applies a binomial amplitude taper in the *horizontal* direction only. The 3D pattern of that arrangement is shown in fig 5.22
4. Because of the way I have implemented `sgarray` it is difficult to apply the amplitude taper in the Vertical dimension (See the Problems section), but one can do it manually. The taper that needs applying is generated by the following matlab code:

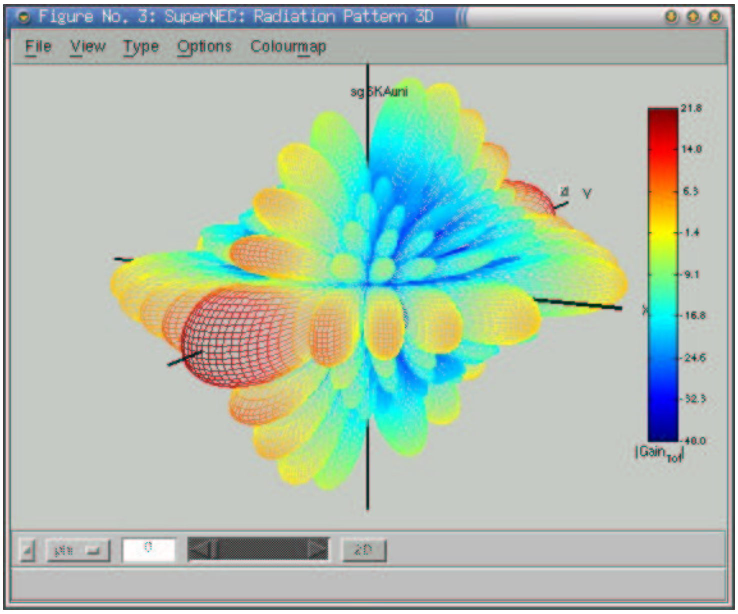

Figure 5.21: 3D pattern of a 10 by 10 Uniform array.

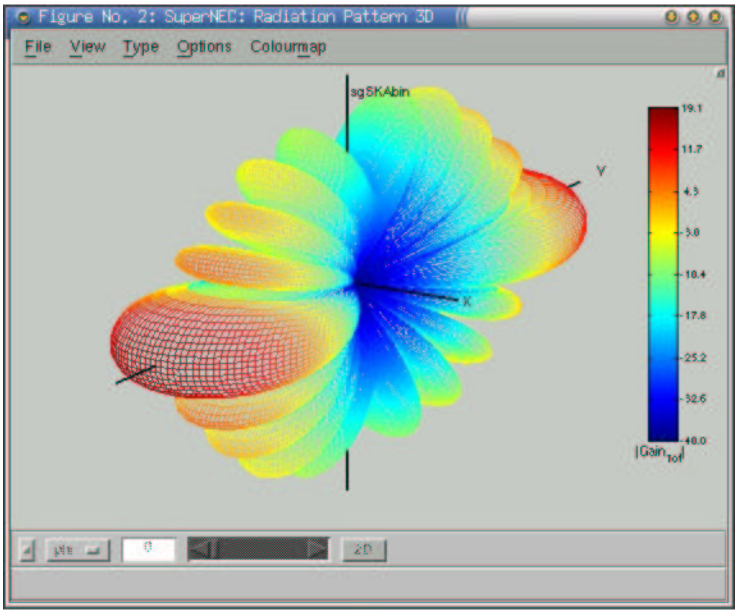

Figure 5.22: 3D pattern of a 10 by 10 short-dipole array with binomial amplitude taper on the horizontal axis only.

```
>> taper = abs(pascal(10,1));
>> taper = taper(10,:)

taper =

     1     9    36    84   126   126    84    36     9     1
```

Thus if we put this into the necessary 2D problem we get an amplitude taper as shown in fig 5.23

1	1	1	1	1	1	1	1	1	1
1	9	9	9	9	9	9	9	9	1
1	9	36	36	36	36	36	36	9	1
1	9	36	84	84	84	84	36	9	1
1	9	36	84	126	126	84	36	9	1
1	9	36	84	126	126	84	36	9	1
1	9	36	84	84	84	84	36	9	1
1	9	36	36	36	36	36	36	9	1
1	9	9	9	9	9	9	9	9	1
1	1	1	1	1	1	1	1	1	1

Figure 5.23: Amplitude tapers required for a 2D binomial (square) array

Thus the fifth and six rows stay as they were and the changes are made to all the other rows symmetrically. There are therefore only two changes to be made to the fourth and seventh rows: to change the two 126's to 84's.

Next complete the 36's "square" etc. This is accomplished by clicking the $<$ of the `Group Level` control in the input editor until `low` is showing, zooming in on the dipole, and selecting the fed segment, and changing the current source value to the one in the above table.

Do Not forget to press the `Modify` button after entering the new current value!

The output is shown in fig 5.24.

Conclusion: It is apparent that amplitude tapering greatly assists in beamforming. As shown in in fig 5.24, a single beam is possible. If the binomial quantities are adjusted *radially* as a function of distance from the centre, the four "skin tags" caused by the corners will also diminish. If various sub-arrays within the array could be formed, it is possible to steer multiple

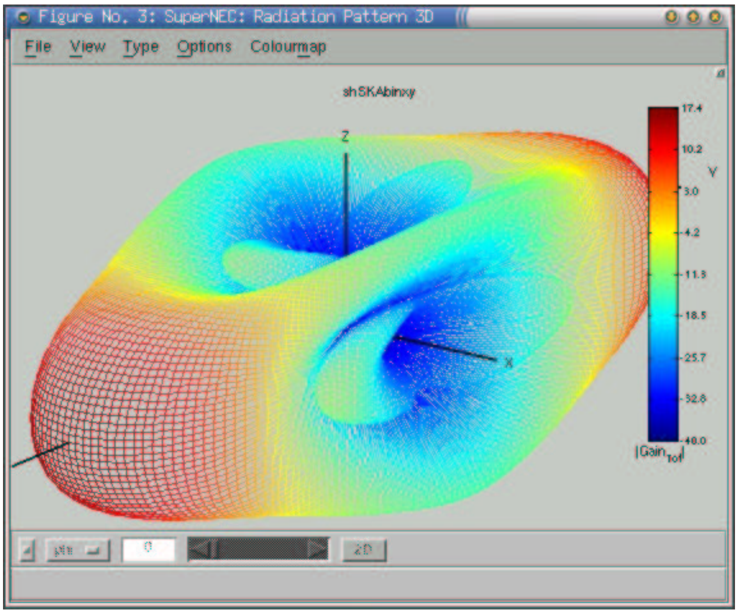

Figure 5.24: 3D pattern of a 10 by 10 short-dipole array with binomial taper applied horizontally and vertically.

beams in different directions—this is the basis of the flat panel RADAR arrays used (for example) in the Patriot II ballistic missile interceptor.

Remember though:

- that to (arbitrarily) be able to modify the *amplitude* and the *phase* for each array element is an expensive exercise, and
- that the array needs *length* in the dimension that it is attempting to compress. Thus a narrow pencil beam is not possible from a simple vertically stacked array with no horizontal width!

5.3 Problems

5-1. Array gain. Determine the gain of a two-stack vertical dipole array using `sgarray`. How does that relate to a single dipole gain (2.16dBi)?

5-2. Array gain. How many dipoles will be required in the vertical array to obtain an extra 3dB over the gain obtained in the previous question.

5-3. Array gain. How many dipoles will be required in a vertical array to obtain 20dBi? Obtain a 3-D pattern!

5-4. Array gain. Construct a 4 by 4 dipole array (16 dipoles in all, fed in-phase). Predict the maximum gain. Note that the array width is needed to compress the azimuth pattern. Plot the radiation pattern and determine the HPBW in the azimuth and elevation planes.

5-5. Beamwidth vs Array Length Prove the formula that relates the required beamwidth to the *length* of the array (using the default spacings)

5-6. Beamwidth vs Array Length Repeat the above, but with double the spacing between the elements.

5-7. Beamwidth vs Array Length Show how you can use a "cross" of short dipoles to compress the gain in two dimensions.

5-8. Phasing. Calculate the progressive phase shift δ required for an 8-dipole horizontal array, spaced $\lambda/2$ apart, in order to achieve the end-fire case. Plot the Azimuth radiation pattern.

5-9. Phasing. For the previous problem calculate the δ required for a beam to be directed at $60°$? For $-60°$?

5-10. Large Array Show the difference between using the "cross" of prob 5.3 and the full 10 by 10 array of short dipoles.

5-11. Large Array Change the weightings of the xy binomial 10 by 10 array to get rid of the "corners" shown in fig 5.24.

5-12. Large Arrays Calculate the progressive phasing required to swing the main beam of the 10 by 10 uniform array $20°$ left, right, up, and down. Implement.

5-13. Creating Assemblies Create a new assembly, based on `sgarray`, that accepts a matrix of current amplitudes, *and phases* for feeding a square array.

5-14. Creating Assemblies Create a new assembly, based on `sgarray`, that can construct a *Circular* array, also with the ability to accept a weighting and phasing matrix. This should give a better binomial performance than problem 5.3.

5-15. Using the new assemblies Create a 12 by 12 matrix with four 4 by 4 sub-matrices, and using binomial tapering, create a four lobed steerable array. If you have access to a Cray, try a 100 by 100 matrix!

Chapter 6

Complex Antennas

This chapter covers the more complex antennas such as helices, Yagi-Uda arrays, Log Periodic Dipole Arrays etc. It is differently structured to the rest of the Study Guide, as it makes no sense to separate the "Theory" from each particular antenna from the illustrative SuperNEC exercises. It is in this chapter that SuperNEC is most "at home": although I have demonstrated many things using it in earlier chapters (in many cases surprising me) SuperNEC is essentially designed to analyse antennas!

6.1 Dipole Arrays

From array theory, it is clear that a collinear array of dipoles is a uniform broadside array. It is almost exclusively used in the broadside mode of operation since the dipoles themselves have maximum directivity in this direction. From the broadside condition this implies that the elements of such an array should always be *in phase*.

6.1.1 The Franklin array

This is an ingenious method of arraying three half-wave dipoles with half a wavelength spacing, and is shown in fig 6.1.

The quarter wave phasing section reverses the current phase by 180° and ensures that all three dipoles are in phase. The gain of this arrangement has been measured at 4 to 5 dBi.

Often, the phase reversal section is a loading coil, which has a sufficient amount of self-capacitance to form a resonant L-C network, which performs the 180° phase shift. Many cellphone car-kits employ this technique, obviously in monopole form. The L-C network to do the phase reversal is, in many cases a physical L-C network, encased in plastic for rigidity.

Exercise 6.1: Franklin array

Purpose: *To illustrate the Franklin array and relative phasing between its sections, as compared to a straight dipole of similar length.*

1. Add an ordinary dipole to the Input Viewer using `Add| Assembly| antennas| sndipole` accepting the defaults.

   Then add a multiple-wire structure for the phase reversal transmission line and the next dipole in the chain: `Add| Assembly| structures| snwires` (Note `snwires` not `snwire`).

   Since we know that the top of the `sndipole` is at `[0 0 0.25]` that is the starting coordinate for the `snwires`, and we build up from there. Thus enter the following set of coordinates into the `Coordinates` entry in the dialogue box: `[0 0 0.25;0 0.25 0.25;0 0.25 0.26;0 0 0.26;0 0 0.76]` paying careful attention to the semicolons. Do the same in the negative z direction, ie `[0 0 -0.25;0 0.25 -0.25;0 0.25 -0.26;0 0 -0.26;0 0 -0.76]`, obtaining the Franklin Array shown in fig 6.1.

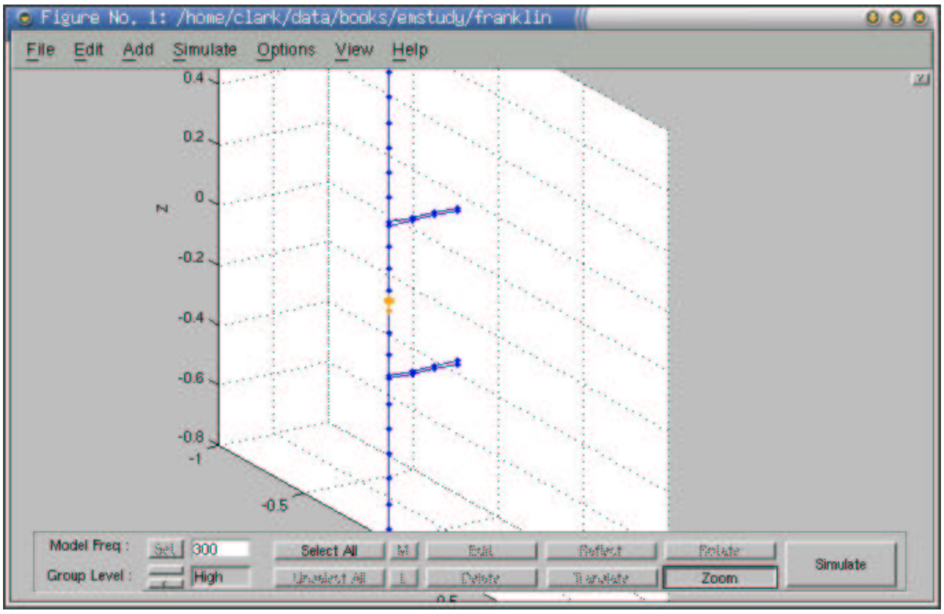

Figure 6.1: Franklin Array of three dipoles, with phase-reversal transmission lines

2. Add an xy and xz plane radiation pattern, using `Edit| Simulation Settings` and simulate. The Azimuth pattern is still omnidirectional, and the elevation plane is shown in fig 6.2. Record the broadside gain achieved in table 6.1.
3. Plot the current distribution on the Franklin Array. It is very clearly seen that the dipole sections exhibit the correct current distribution, as required by array theory.

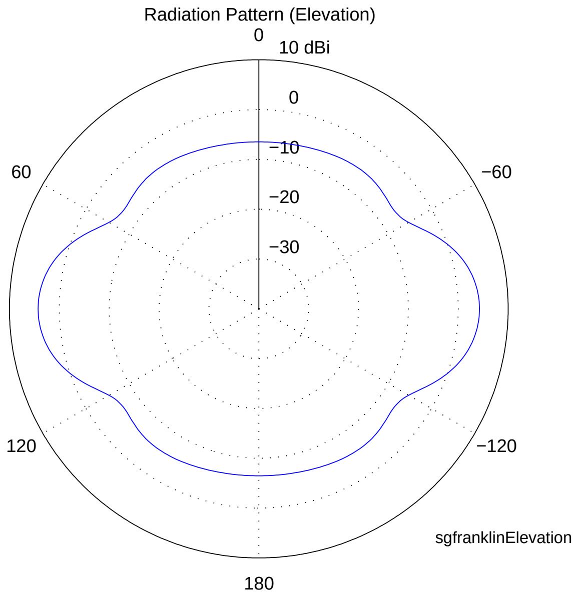

Figure 6.2: Elevation plane pattern of a 3-dipole Franklin Array

4. By way of contrast, click `Select All` and `Delete` in the Input Viewer, and add a long dipole with `Add| Assembly| antennas| sndipole` and change the ends to -0.75 to +0.75. Plot the Radiation Pattern, recording the broadside gain in table 6.1.

Table 6.1: Broadside gain of a franklin array, with and without phase reversal transmission lines.

Case	Broadside gain (dBi)
dipole	2.16
3 dipole Franklin	
long dipole	

It is instructive to pull up the output file, and search for the string `- - - CURRENTS AND LOCATION - - -` under which you will see something like table 6.2

Showing very clearly that the phasing in the top and bottom dipoles is wrong, and hence the weird pattern. It is more effectively an end-fire array rather than a broadside array.

5. Use `File| Open` to retrieve your saved Franklin array again, and set the model frequency to 400MHz, remembering to click `Set` to re-segment the model. Set up a frequency sweep under `Edit| Simulation Settings` from `[200:400]` and add a single-point radiation pattern in the xy plane, changing the `Phi` entry to `[0,306,1]`. Remember to press the `Add` button!

   Simulate and plot the Gains as a function of Frequency.

Table 6.2: Current phase on $3\lambda/2$ dipole.

PHASE
150.182
151.881
154.066
157.473
165.640
-109.439
-38.229
-29.233
-24.192
-29.233
-38.229
-109.439
165.640
157.473
154.066
151.881
150.182

6. Click `Select All` and `Delete` to clear the input viewer and use `Add| Assembly| antennas| sndipole` to add a default dipole. Simulate and overlay the gain plot, as shown in fig 6.3

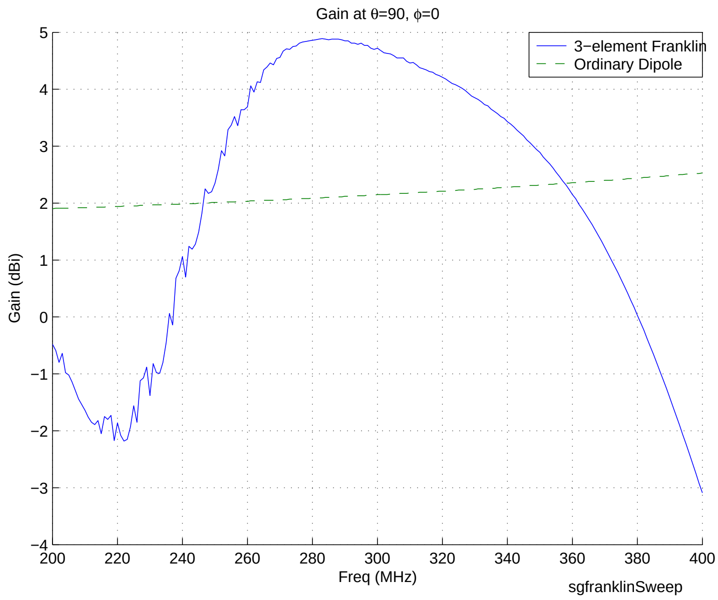

Figure 6.3: Gain as a function of frequency for a Franklin Array as compared to a dipole

Using markers, record the 3dB Gain-bandwidth of the Franklin Array in table 6.3

Conclusion: The Franklin Array is an ingenious method of fulfilling the re-

Table 6.3: Gain-bandwidth of a 3-section Franklin Array

	3dB gain BW
3-section Franklin	

quirements of array theory (fed in-phase) in a simple way. Note that if the phase reversal does not happen the pattern is severely distorted away from the broadside.

The gain bandwidth of the array is limited, since as you move away from the centre frequency, the phase reversal no longer works correctly, and a severe plummeting of the gain occurs at the high frequency end.

6.1.2 Series fed collinear array

It is often useful to have an end-fed array to avoid cables dangling about, and the Franklin array above can also be fed at the bottom, if that is converted to a monopole length and fed against a ground plane. An example of a series-fed collinear array is shown in figure 6.4, which uses a different principle than the Franklin Array, but is exceptionally attractive (and very cheap to manufacture).

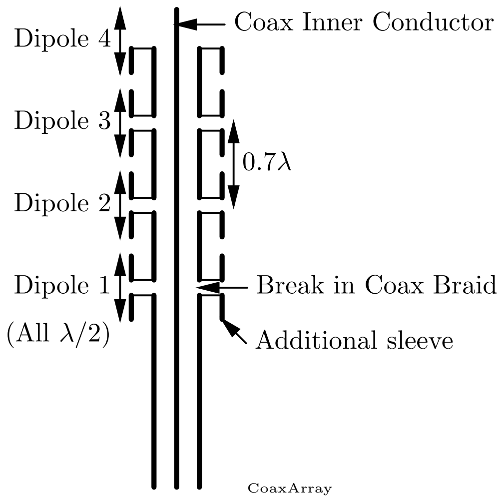

Figure 6.4: A Series Fed Four Element Collinear Array

Essentially, you take a length of coax cable, take a quarter-wave length of the braid off, exposing the inner conductor. This is the top of Dipole 4 in fig 6.4. Move down the coax by 0.7λ, cut the braid all round. Repeat! Strip the braid from another piece of slightly larger diameter coax and cut seven sleeves, each a quarter wavelength. Slip them over the PVC insulation of the antenna coax and solder as shown by the horizontal lines. Cast in resin.

This arrangement is thus equivalent to a 4 dipole array with a 0.7 wavelength spacing. The value of 0.7 wavelength spacing results in one wavelength electrical length (0.7/0.66) between the excitation slots and so ensures *in-phase* operation. The gain of this antenna is about 8.5 dBi. Usually such antennas are mounted in a fibreglass radome to give them mechanical rigidity. Unfortunately there is no DC path to earth, and lightning protection is a problem with this array in South African conditions.

The year the SA cell network first rolled out, in June or so, these arrays were used all around Gauteng, come September, they were replaced!

This antenna *can* be modelled in SuperNEC, but it is left as an "exercise to the reader"!

6.1.3 Collinear folded dipoles on masts

From array theory, it is fairly easy to calculate the gain of a collinear array of dipoles in free space. The effects of a mast will distort the azimuth pattern somewhat however.

Folded dipoles are usually mounted about a quarter of a wavelength from the mast to yield some gain from the mast reflection and to ensure practical boom-lengths and feed harnesses.

Exercise 6.2: Folded Dipoles on a Mast

Purpose: *To illustrate the effect of a mast on antenna performance, and to show that "Omnidirectional" azimuth coverage is extremely hard to get!.*

1. Generate a 2m mast, centred at the origin by `Add| Assembly| structures| snmast` changing the defaults so that the `Location` is at `[-0.05 0 -1]`, the `Height` is 2m, the `Side length` is 0.1m and uncheck the `Provision for ground` boolean.
2. Add a Folded Dipole using `Add| Assembly| antennas| snfdipole` and change the defaults such that its `Location` is at `[-0.25 0 0]`, and it Orientation at `[90 0 90]`, as shown in fig 6.5
3. `Edit| Simulation Settings` and add an xy radiation pattern, simulate, plot and record the forward and backward gains in table 6.4

Table 6.4: Influence of a thin mast on a folded dipole's pattern.

	Gain (dBi)
Forward (180°)	
Backward (0°)	

4. A common trick in this kind of Base Station (typically in the 400MHz region for Trunking purposes) is to vary the dipoles, each offset 90° around the mast, vertically spaced about $3/4\lambda$ apart. If you analyse this in terms of Array Theory, it works quite well in azimuth, but is terrible in Elevation.

But you don't care about Elevation in Land Mobile use, anyway!

Put the Folded Dipoles around the mast using `Add| Assembly| antennas| snfdipole` with the following specifications:

- `Location: [0 0.25 0.75]`, `Orientation, [90 0 0]`.
- `Location: [0.25 0 1.5]`, `Orientation, [90 0 270]`.

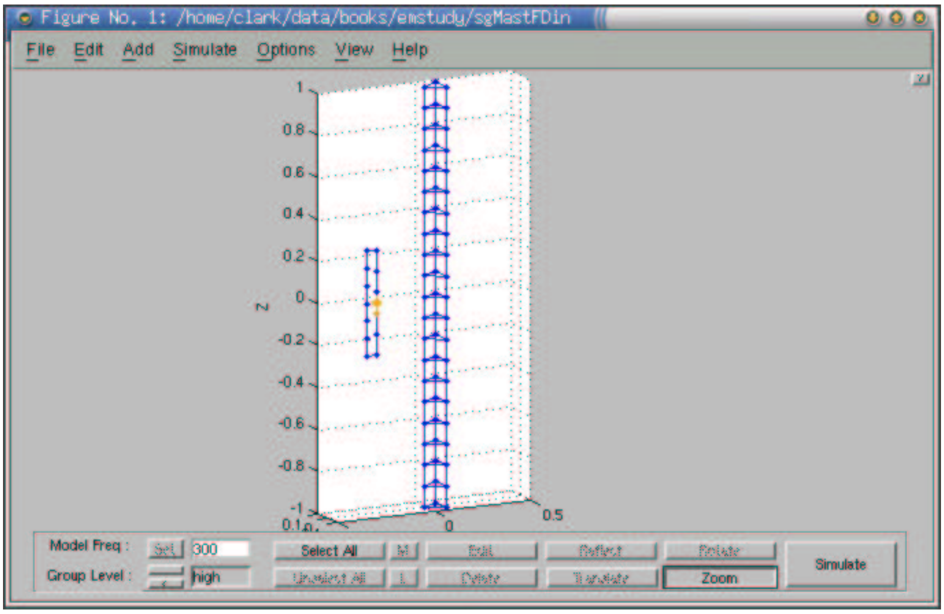

Figure 6.5: A folded dipole $\lambda/4$ away from a mast (0.1m sides)

- `Location: [0 -0.25 2.25]`, `Orientation, [90 0 180]`.

Change the mast length to 4m, located at `[-0.05 0 -0.875]`. You should see something like fig 6.6 and fig 6.7.

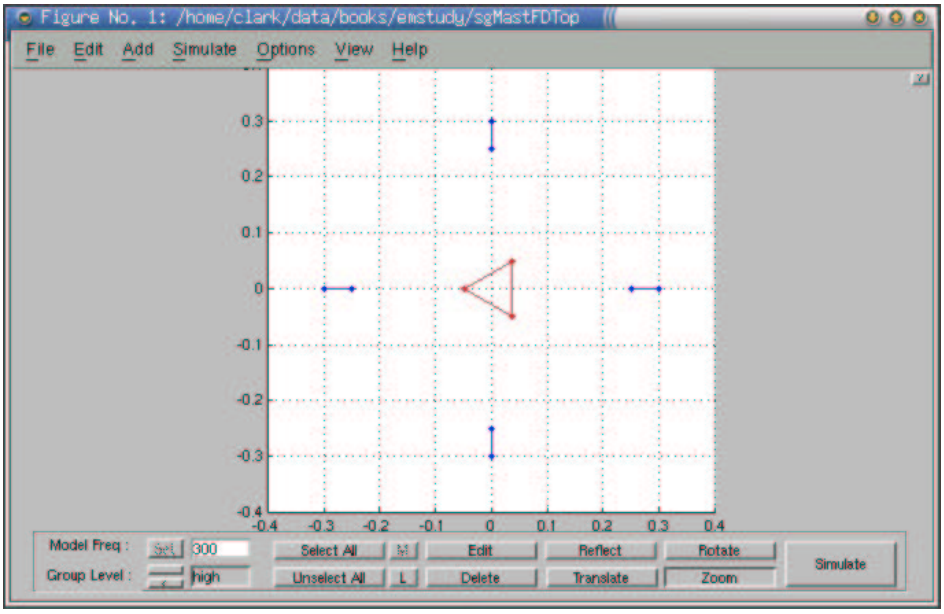

Figure 6.6: Mast with four folded dipoles, seen from the top.

5. Plot the radiation pattern and record the best and worst Azimuth gain in

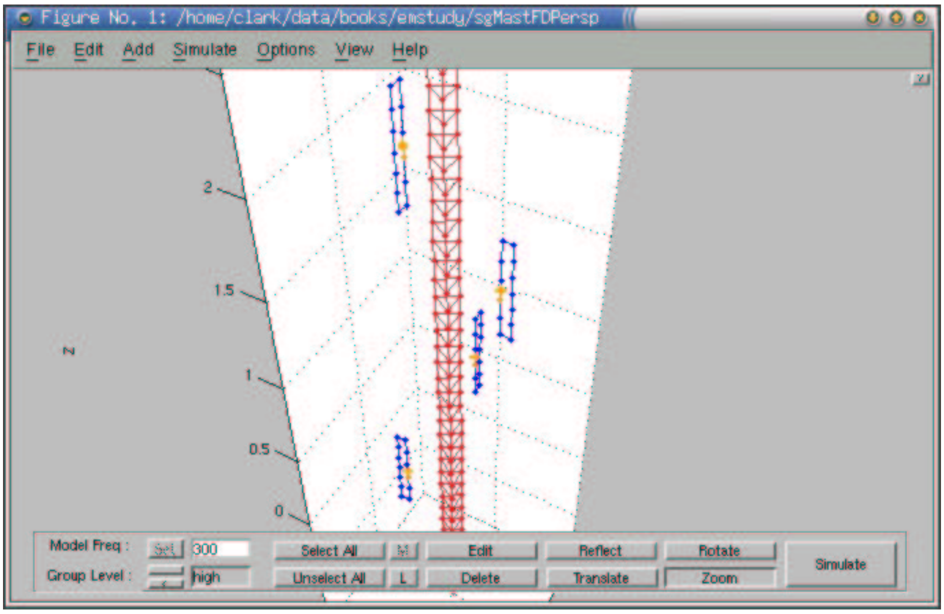

Figure 6.7: Mast with four folded dipoles, in a perspective view

table 6.5

Table 6.5: Azimuth gain variation of an offset Four-stack.

	Gain (dBi)	Angle ($\circ$)
Best gain		
Worst gain		

6. Killing the 2D pattern and putting in a 3D one results in fig 6.8.

Conclusion: The trouble with a mast is that it gets in the way! It has been shown that even the feeder cable produces a similar effect. .

Thus the only truly "omni" is the series-fed collinear

A 6dB variation in gain is seen when a mast is present. A clever application of array theory is to keep the folded dipoles a quarter wavelength away from the mast, but stagger them going up the mast. in this way, the inevitable nulls that must occur from array theory tend to be at some angle that is not on the Azimuth plane. Since most applications want an Azimuthal "omni", this works very nicely, but from an elevation perspective fig 6.8 is rather a mess!

6.2 Yagi-Uda array

If you ask a 6 year old child to draw an "aerial" it is likely to be a Yagi-Uda array. This indicates the popularity of this antenna and not without reason. A "Yagi" antenna is probably the simplest, cheapest and most effective medium gain antenna available, and is found on every rooftop!

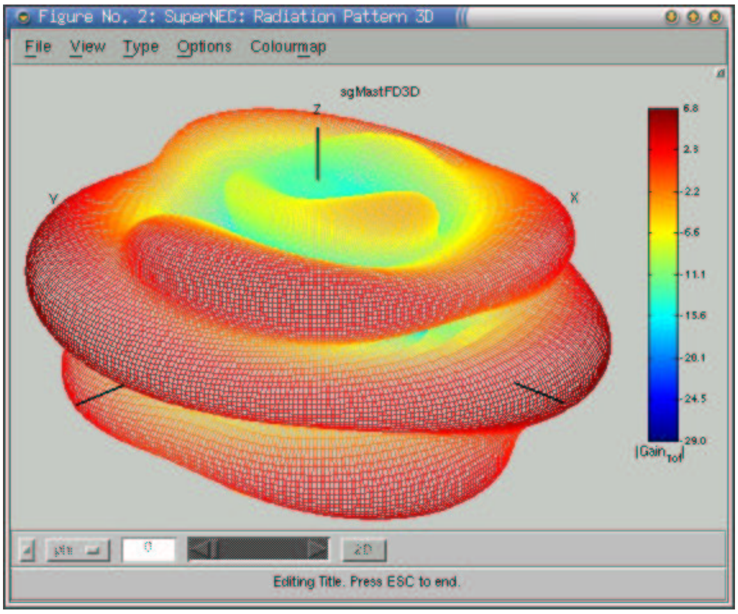

Figure 6.8: Three Dimensional pattern of an offset Four-stack.

The Yagi-Uda antenna, shown in fig 6.9 on page 97 was invented in 1926 by Dr. H Yagi and Shintaro Uda Yagi [1928].

Since then numerous reports on this antenna have appeared in the literature—one of the most noteworthy is the study done by Viezbicke [1976] of the U. S. National Bureau of Standards (NBS). He optimised the gain of a number of Yagi antennas and investigated the effect of the boom and element length on the performance of the antenna. The NBS experimental findings were later confirmed during an excellent series of articles on Yagi antenna design by James Lawson in the Ham Radio magazine (1979–1980). These articles were later combined in a book by the ARRL Lawson [1986], which is the best practical Yagi-Uda design text available today.

Uda was Yagi's postgraduate student. Although Uda published many papers in Japanese (with Yagi as a co-author), Yagi's publication was the first in English journals (without Uda as a co-author). Hence "Yagi-Uda", not just "Yagi"!!

6.2.1 Pattern formation and gain considerations

The antenna usually has only one *driven* element, usually a folded dipole; the other elements are not directly driven, but are *parasitic*, obtaining their current via mutual coupling. The spacing between elements is approximately a quarter wavelength. The reflector is slightly longer than required for resonance and is thus *inductive* (current phase *retarded*). The directors are shorter than resonance and therefore exhibit a *capacitive* reactance and hence a phase *advance*. The overall structure therefore has a progressive phase in the forward direction and it behaves like an endfire array.

As a general rule of thumb the gain is directly proportional to boom length for well designed Yagi's. In other words, a 3 dB gain increase is obtained by doubling the boom length. The number of elements *per se* is not the determining factor.

However, remember the "Law of Diminishing Returns": with each doubling, something less than 3dB is added, by virtue of decreased current induction in the far elements.

6.2.2 Design

A baseline Yagi-Uda design is given by Lawson [1986] and is given by table 6.2.2

Yagi-Uda

Yagi Design Details (All dimensions in wavelengths)

Boom length	0.4	0.8	1.2	2.2	3.2	4.2
Reflector	0.482	0.482	0.482	0.482	0.482	0.475
Reflector spacing	0.2	0.2	0.2	0.2	0.2	0.2
No of directors	1	3	4	10	15	13
Director	0.442	0.427	0.424	0.402	0.395	0.401
Director spacing	0.2	0.2	0.25	0.2	0.2	0.308
G(dBd) (Lawson)	7.1	9.2	10.2	12.25	13.4	14.2
Driven (SN)	0.426	0.421	0.417	0.423	0.435	0.434
$R_{\rm in}$	8.6	20.4	19.0	43.4	55.6	44.5
Driven FD (SN)	0.389	0.382	0.378	0.382	0.396	0.396
$R_{\rm in}$	34.1	76.1	72.1	158.3	202.9	166.2
SuperNEC gains(dBi)	9.1	10.5	10.8	12.3	12.7	13.3

The length of the driven element can be chosen for the optimum match condition since it does not affect gain operation much. As an example, the SuperNEC determined resonant lengths and input impedances are shown in the above table. Driven (SN) refers to the SuperNEC derived dipole driven lengths, and Driven FD (SN) is the SuperNEC derived folded dipole driven lengths. All diameters are 0.008λ, and the folded dipole separation is 0.05λ. The gain as calculated by SuperNEC is shown in the last line, in dBi. As can be clearly seen, the Lawson gain figures are optimistic for the longer arrays.

Exercise 6.3: Yagi-Uda array

Purpose: *To illustrate the properties of the Yagi-Uda array: its gain and impedance bandwidth, and pattern characteristics.*

Yes, I know purists will insist on `snyagiuda`!

1. Construct a standard 10-director (12-element) Lawson Yagi-Uda as in table 6.2.2 by `Add| Assembly| antennas| snyagi` , therefore changing the defaults:
   `Yagi Element Spacing` to
   `[0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2 0.2]` (ie 11 gaps);
   `Yagi Element Lengths` to
   `[0.482 0.423 0.402 0.402 0.402 0.402 0.402 0.402 0.402...`
   `0.402 0.402 0.402]` and
   `Yagi Element Radii` to
   `[0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01...`
   `0.01 0.01]`

It will look something like fig 6.9.

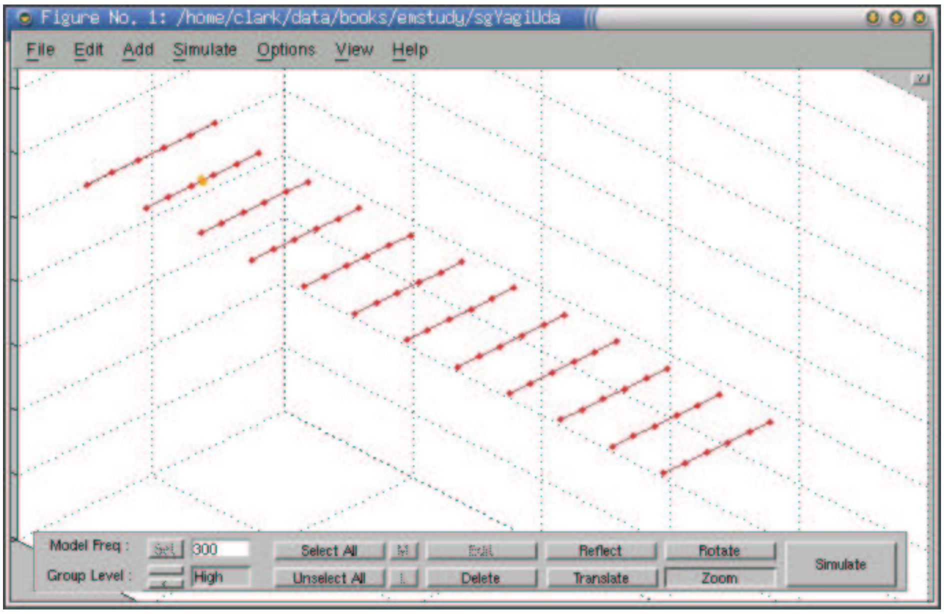

Figure 6.9: A 12-element Yagi-Uda Array

2. Using `Edit| Simulation Settings` add an xy and xz plane radiation pattern, simulate, and plot. Record the Half-Power BeamWidths (HPBW) in the theta and phi planes in table 6.6

Table 6.6: Half-Power BeamWidths of a 12 element Yagi-Uda array.

	3dB Beamwidth (°)
Azimuth (ϕ)	
Elevation (θ)	

3. Now for the Swept Frequency analysis of the antenna, use `Edit| Simulation Settings` to delete the two Radiation Patterns, and add an xy plane pattern request, changing the `Phi` entry to `[0,360,1]` to obtain only the boresight maximum gain point. Change the frequency to `[250:350]`, and the `Model Frequency` to 350MHz. Exit the Output Viewer, simulate, and plot the gain versus frequency as shown in fig: 6.10. Record the 3dB gain bandwidth as well as the 2:1 impedance bandwidth in table 6.7.

Table 6.7: Gain and Impedance bandwidth of a 12 element Yagi-Uda array.

	Bandwidth	Percent
Gain (3dB)		
Impedance (2:1 VSWR)		

Conclusion: Note that the HPBW's differ in the phi and theta planes. If you look end-on at a Yagi-Uda, you will see why: tilting it up and down, you

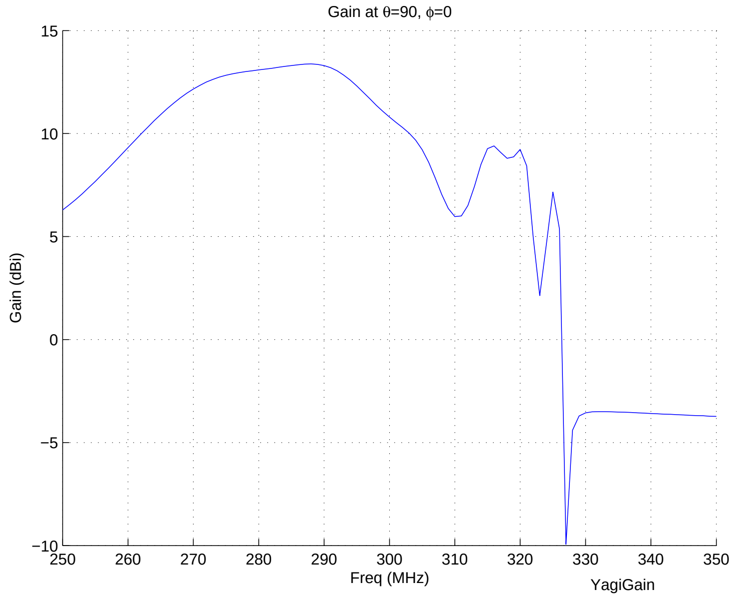

Figure 6.10: Gain versus frequency for a 12 element Yagi-Uda array

still see the omnidirectional pattern of the individual dipole doughnuts; tilting the Yagi-Uda left and right means that you are seeing more of the doughnut hole! Hence the HPBW in the theta direction is larger than that in the phi direction.

Note that the gain bandwidth is much wider than the impedance bandwidth: this is again a generalisation for a resonant antenna structure. If only we could match it over a wider impedance range, the gain would still be there! See problem 6.5.

6.3 Log Periodic Dipole Array

The LPDA, first proposed by Isbell [1960], is a truly frequency-independent antenna and probably the most popular broadband array. The term *true frequency independence* in this instance implies *pattern* and *input impedance* constancy. These antennas are used successfully in applications ranging from HF to microwaves. Carrel [1961] developed a particularly straight-forward design procedure for these antennas which has ensured their success.

Carrel disregarded the effects of the characteristic impedance of the antenna boom/transmission line and also the thickness of the elements. Peixeiro [1988] presents a more complete design technique catering for these defects, but which still incorporates a large number of the features introduced by Carrel. The general form of an LPDA antenna is shown in figure 6.11.

The array is fed at the small end of the structure, and the maximum radiation is toward this end. The lengths of the dipoles and their spacing are varied such

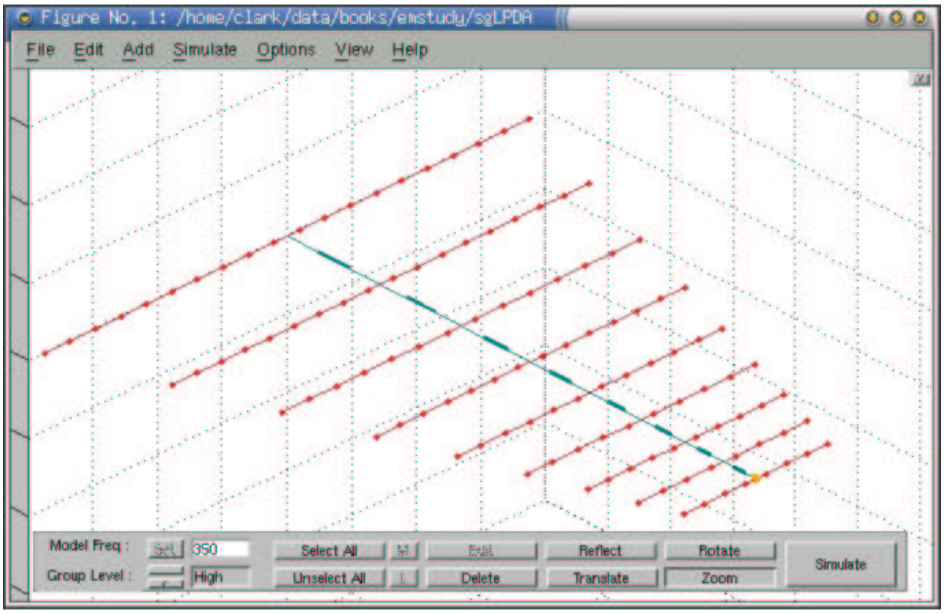

Figure 6.11: The log-periodic dipole array

that these dimensions bear a *constant ratio* to each other—regardless of the position on the antenna (except at the two ends). ie The antenna *scales* itself: the *scale factor*, τ, is one of the design parameters.

For any dipole in the array, its length(L_n) is related to the next (smaller) dipole(L_{n+1}) by τ. The distance between the dipoles (d) is similarly *scaled*:

$$\boxed{\tau = \frac{L_{n+1}}{L_n} = \frac{d_{n+1}}{d_n}} \tag{6.1}$$

It is apparent that these conditions cause the ends of the dipoles to trace out an angle, 2α. When the antenna is fed from the small end with a frequency that is much too low for the short dipoles to resonate, these elements will absorb very little power (hence they will radiate very little power too). The phase of the current is mechanically changed by 180 degrees between these electrically short elements. The radiation any of these will produce will therefore be cancelled by the out-of-phase radiation of adjacent elements.

Once a portion on the antenna has been reached where the dipoles are resonant and electrically further apart these dipoles will absorb most of the energy from the transmission line and radiate it. This part of the antenna is called the *active region*. If the frequency is increased, this active region will simply move towards the small end of the antenna. This explains the frequency independence of the antenna for frequencies where the active region is not at one of the two ends.

The directional property of the antenna is due to the elements in front of the active element being shorter than resonance and therefore capacitive—and act as directors. Similarly, elements behind (towards the large end) act as reflectors—giving the antenna a endfire beam towards the small end.

The *space* factor *sigma* provides the spacing of the array and is given as:

$$\boxed{\sigma = \frac{d_n}{2L_n}} \tag{6.2}$$

According to Peixeiro, the optimum τ, σ pairs are as listed in table 6.8

Table 6.8: Optimum LPDA τ, σ pairs for different gain values.

Gain (dBi)	8	9	10	11	12
τ	0.860	0.898	0.926	0.950	0.960
σ	0.170	0.189	0.200	0.213	0.220

Exercise 6.4: Log Periodic Dipole Array

Purpose: *To investigate the properties of the Log Periodic Antenna.*

1. Construct a standard LPDA using `Add| Assembly| antennas| snlpda`, ($\tau = 0.86, \sigma = 0.1$) then `Edit| Simulation Settings` and change the frequency to a sweep `[50:300]` and add a single point xy plane radiation pattern, changing the `Phi` entry to `[0,360,1]`.

   The gain is shown in fig 6.12 and the VSWR in fig 6.13.

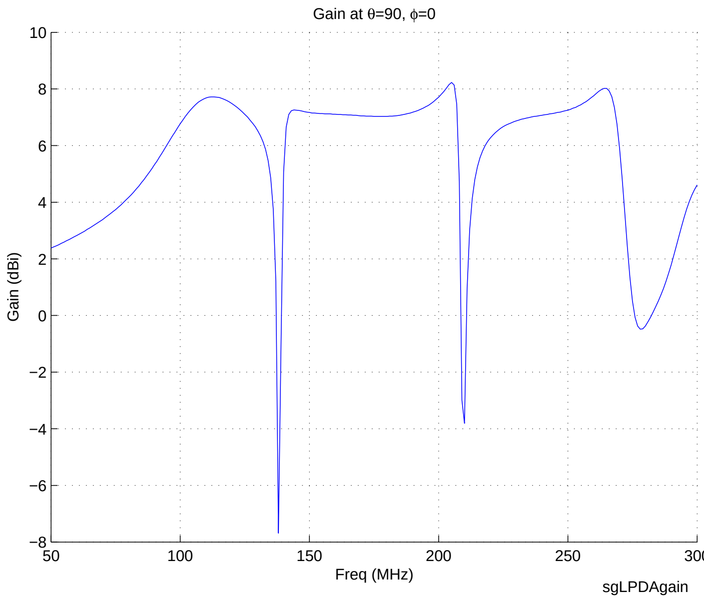

Figure 6.12: The Gain bandwidth of the standard LPDA.

2. Record the frequencies at which the gain and VSWR anomalies occur. In the Output Viewer, select the `Currents/Charges` tab, and select `All` frequencies and plot. The current distribution at 50MHz is shown in fig 6.14

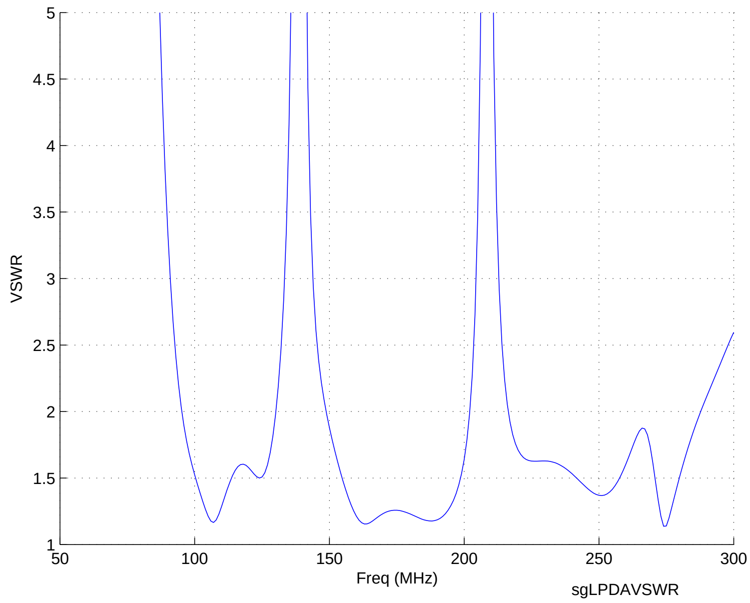

Figure 6.13: The VSWR bandwidth of the standard LPDA.

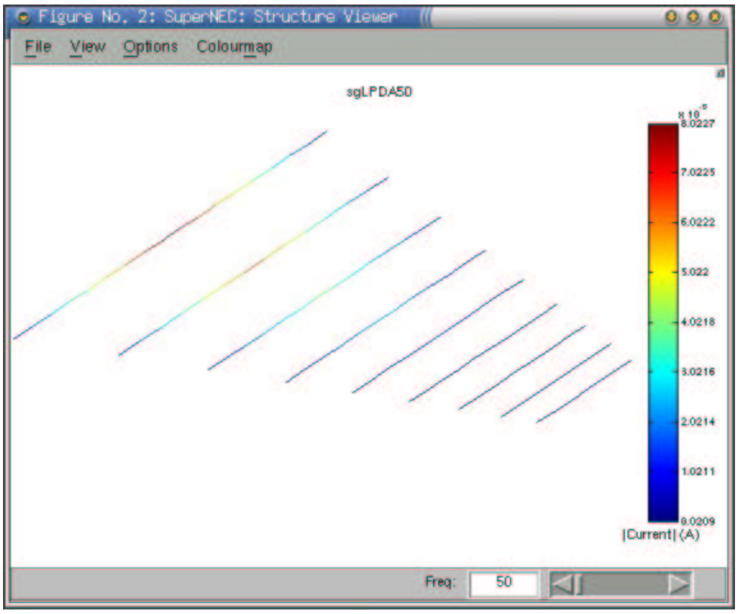

Figure 6.14: The Current Distribution on the LPDA at 50MHz.

Clearly, the large end of the structure is the active region.

3. Click on the slider bar in the frequency scrollbar at the bottom of the Current Viewer, and move it along, watching as the active region moves higher up the antenna as the frequency increases. Fig 6.15 shows the active region at 170MHz.

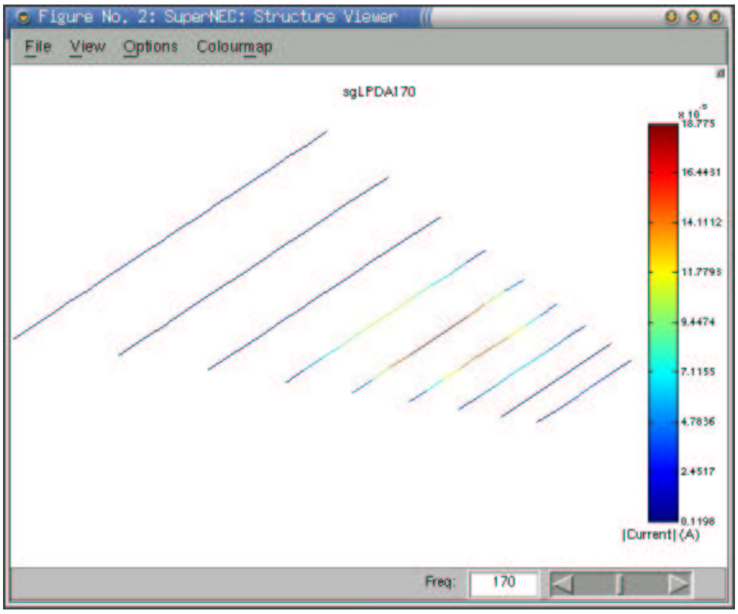

Figure 6.15: The Current Distribution on the LPDA at 170MHz, showing a shifted active region

4. Now return to the points of gain and VSWR anomalies. In my particular simulation (yours may differ), 138 and 210MHz were the worst dips. Entering 210MHz into the frequency entry in the Current viewer, produces fig 6.16.

Conclusion: The LPDA is an example of a multi-resonant structure. Its impedance bandwidth exceeds its gain bandwidth, and the gain “truncates” before the resonance of the “last element”.

It suffers from self-generated “Franklin-array” induction on the longer elements, obviously without the phase reversal properties of a proper Franklin array. From array theory, this obviously produces wild variations in the forward gain at these (very narrow) frequencies. In fig 6.16 the main active element is the third dipole, but clearly seen on the sixth dipole are *two* regions of activity, obviously without phase reversal! Two regions of slightly less current density are visible on the seventh dipole. By looking carefully, you will see *three* regions on the longest dipole. This particular antenna doesn’t stand a chance at 210MHz!

I once designed a rotatable 12–30 MHz LPDA for a customer, and he was “very happy” with the results: “but I can’t get frequency xx.yyy, but I can get xx.yzz”. I patted him on the back, and patiently explained that I was an academic and knew that the LPDA was “frequency-independent”. In those days, we did simulations at 5MHz intervals to save computation time: the gain “glitches” were simply not picked up. It took SuperNEC and 1MHz intervals to prove me, the all-wise academic, wrong, and the old codger, right!

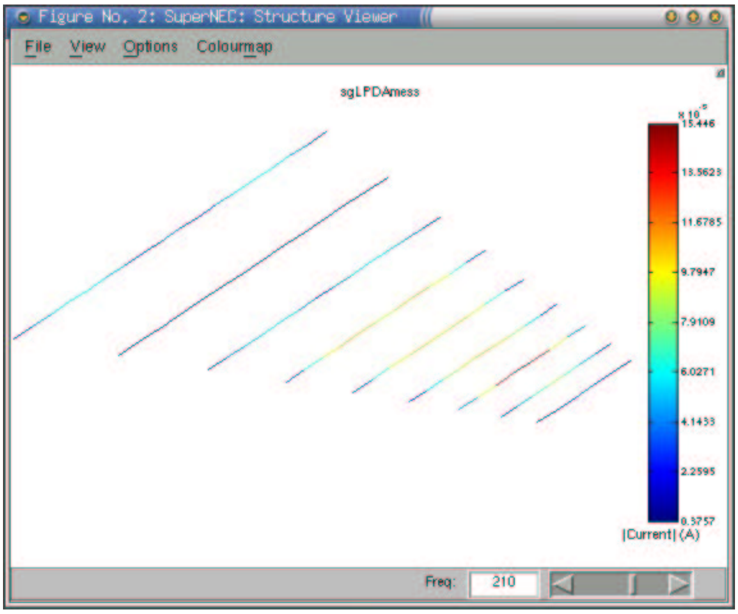

Figure 6.16: Messy Current Distribution at 210MHz. Note the multiple resonance on several dipoles, even the longest one!

6.4 The Axial-mode Helix

In 1946, J.D Kraus attended a physics lecture in which a helical structure was used to guide an electron beam in a travelling wave tube. He asked the lecturer about the possibility of using the helix to radiated electromagnetic wave into space, to which the answer was an emphatic NO! Nevertheless, Kraus went home and started to experiment with the structure. As he suspected (or was it to his amazement), the helix showed good promise as an antenna.

When the diameter D and the spacing S are large fractions of a wavelength, the operation of the helical antenna changes considerably from the normal-mode behaviour, and it behaves as an endfire array of loops and the pattern has a main beam in the axial (end-fire) direction.

To excite the axial mode of operation, the circumference of the helix, $C(= \pi D)$, must be in the range

$$0.8 \leq \frac{C}{\lambda} \leq 1.15 \tag{6.3}$$

with a circumference of 1 near optimum. The spacing between turns, S must be about $\lambda/4$ and the pitch angle, α in the range:

$$12^\circ \leq \alpha \leq 14^\circ \tag{6.4}$$

where $\alpha = \arctan(S/C)$

Most often the antenna is used in conjunction with a ground plane, whose diameter is at least $\lambda/2$. The number of helix turns, N, should be more than 4.

Intuitively, the circularly polarisation comes about since "opposite" sides of the helical turn are 180° out of phase, hence providing the **E**-field vector in that plane. Also, referring back to the Yagi-Uda array where the directors are about 0.2λ apart, in order to capacitively "suck" the wave forward, the turns are about 0.21 to 0.25 λ apart (For a C/λ of 1).

Original Kraus design

During the years 1948-1949, Kraus empirically studied the helical antenna and published the following findings (assuming $0.8 \leq C/\lambda \leq 1.15$; $12° \leq \alpha \leq 14°$; and $n \geq 4$):

- The radiation pattern of a helix is predominantly cigar shaped and has a maximum gain given by:

$$G = K_G \left(\frac{C}{\lambda}\right)^2 \left(\frac{NS}{\lambda}\right) \tag{6.5}$$

  where K_G is the gain factor, originally 15, but later reduced to 12 by [Kraus, 1988, pg284]

- The Half Power Beamwidth (HPBW) is given by:

$$\text{HPBW} = \frac{K_B \lambda^{3/2}}{C\sqrt{NS}} \tag{6.6}$$

  where K_B is the beam factor, which is about 52, derived from the standard approximation on beamwidths:

$$G = \frac{41000}{\text{HP}_\theta \text{HP}_\phi} \tag{6.7}$$

  Since the beam is generally circularly symmetric, $\text{HP}_\theta = \text{HP}_\phi = \text{HPBW}$:

$$\text{HPBW} = \frac{\sqrt{41000/K_G}\,\lambda^{3/2}}{C\sqrt{NS}} \tag{6.8}$$

  where $\sqrt{41000/K_G} = K_B$, the beam factor.

- The input impedance is nearly resistive and is given by:

$$R = 140 \left(\frac{C}{\lambda}\right) \quad \Omega \tag{6.9}$$

- The beam is circularly polarised.

King and Wong design

King and Wong [1980] performed a study which involved varying the parameters of a uniform helix and measuring the electrical performance of the structure. They found that the expressions derived by Kraus tended to overestimate the performance of the antenna. Their results are summarised (empirically) as follows:

$$G = 8.3 \left(\frac{C}{\lambda}\right)^{\sqrt{N+2}-1} \left(\frac{NS}{\lambda}\right)^{0.8} \left(\frac{\tan(12.5)}{\tan\alpha}\right)^{\sqrt{N}/2} \tag{6.10}$$

When comparing this result to that published by Kraus, the gain factor K_G is between 4.2 and 7.7 (compared to Kraus's reduced estimate of 12). The beamwidth factor, K_B, is therefore between 61 and 70 (compared to 52).

Please note:

- The revised factors are valid for antennas with $0.8 \leq \frac{C}{\lambda} \leq 1.2$.
- King and Wong also note that the Kraus original factors depend on other design parameters of the helix and are only constant for helices with approximately 10 turns. The revised factors do not suffer from this limitation.

In addition,

- The peak gain of a helix occurs when $\frac{C}{\lambda} = 1.155$ for $N = 5$; and for $\frac{C}{\lambda} = 1.07$ for $N = 35$.
- Since the beam is circular, HPBW$_\theta$=HPBW$_\phi$=HPBW. The Gain-HPBW^2 product was found to be significantly less than 41 000, and lies in the range of 18 000 to 31 500. They note that:
  - $G \times \text{HPBW}^2 = 18\,000$ for $N = 35$ and $0.75 \leq \frac{C}{\lambda} \leq 1.1$
  - $G \times \text{HPBW}^2 = 31\,000$ for $N = 5$ and $0.75 \leq \frac{C}{\lambda} \leq 1.2$
  - The smaller the pitch, the larger the gain-beamwidth product.
- The gain bandwidth of the helix is presented as:

$$\frac{f_H}{f_L} \approx 1.07 \left(\frac{0.91}{G/G_{\text{peak}}}\right)^{\frac{4}{3\sqrt{N}}} \tag{6.11}$$

  where the subscript L refers to the lower frequency, and H the higher; G/G_{peak} is 3dB or 2dB etc according to preference (usually want the 3dB point).

  - Note that the bandwidth decreases as the axial length/ gain/ number of turns increases.
  - The bandwidth is approximately 42% for a helix of $N = 5$; and approximately 21% for $N = 35$.
- The impedance bandwidth (2:1 VSWR) is typically 70%. The input impedance of the helix (with $C/\lambda = 1$) is about 140Ω, almost purely resistive. However, if the last quarter-turn of the helix is made parallel to the ground plane, it creates a quarter-wave transformer, which allows matching down to 50Ω. Since a frequency-selective device has now been introduced, the 70% impedance bandwidth drops to about 40%. This can be ameliorated by *tapering* the matching section in the usual way.

Exercise 6.5: The Helical Antenna

Purpose: *To investigate the Stalwart of Deep-Space antennas: The Helix.*

1. `Add| Assembly| antennas| snhelix` will pull up a standard helical antenna, but click on the `Peripheral Feed` checkbox as shown in fig 6.17.

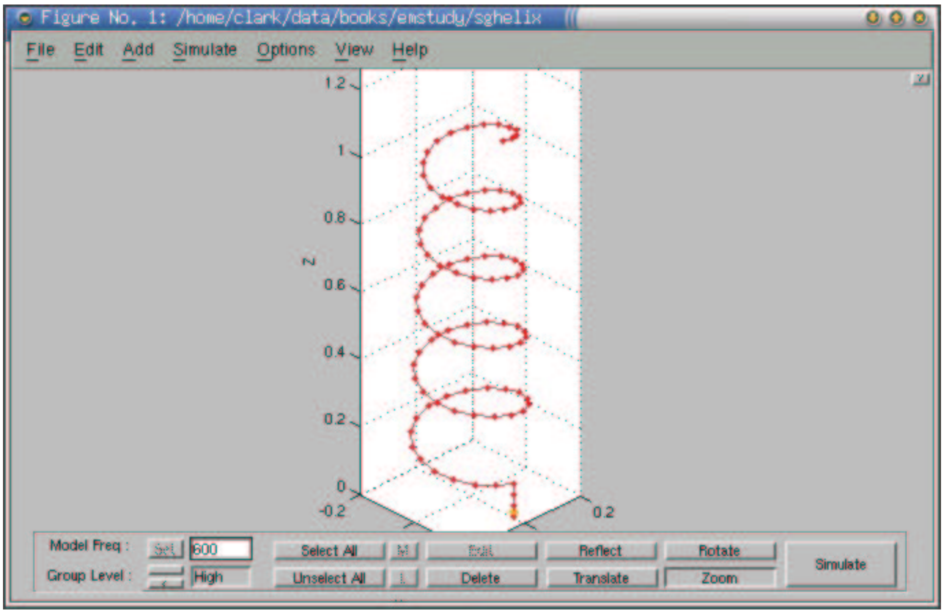

Figure 6.17: A Standard Axial-Mode Helix.

2. Remember that the helix must be fed against a Ground, use `Add| Ground` and choose the `Type` of ground to be `Perfect`.
3. Using `Edit| Simulation Settings` to add a xz radiation pattern changing the `Theta` from `[0,360,361]` to `[-180,180,361]`. Plot the radiation pattern as shown in `fig:sgHelixHPBW`, and using markers, record the Half-Power Beamwidth in table 6.9. Compare the SuperNEC HPBW with the theoretical predictions.

Table 6.9: Half-Power BeamWidths of a 5 turn Helix.

	3dB Beamwidth (°)
xz plane	
yz plane	

4. For the frequency sweep, exit the Output Viewer, set the model frequency to 600MHz, `Edit| Simulation Settings` and set the `frequency` to `[100:600]`, add a yz plane Radiation Patter request limiting to one point in the vertical direction by changing `Theta` from `[0,360,361]` to `[0,360,1]`.

   Plot the gain bandwidth as shown in fig 6.18 and the impedance band-

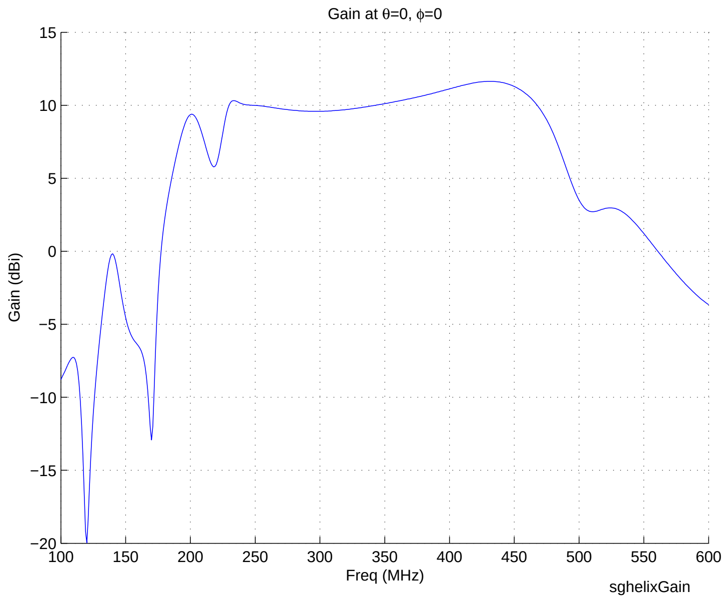

Figure 6.18: The Gain versus frequency of a Standard helix

width, when normalised to `Options| Zo...` of 140 Ω, as shown in fig 6.19 Using markers, record the gain and impedance bandwidths in table 6.10

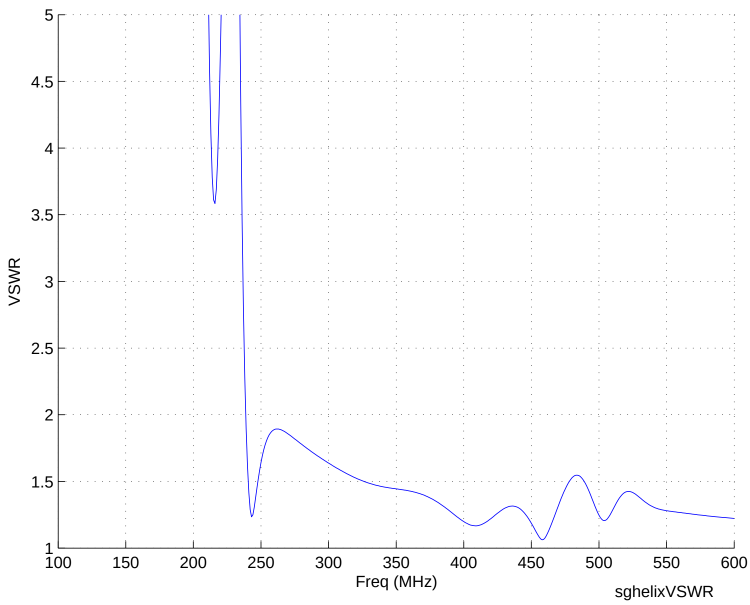

Figure 6.19: The Impedance bandwidth of a Standard Helix, normalised to 140Ω

Conclusion: Note that the impedance bandwidth of a helix is wider than the

Table 6.10: Gain and Impedance bandwidth of a Helix.

	Bandwidth	Percent
Gain (3dB)		
Impedance (2:1 VSWR)		

gain bandwidth—evidence that the antenna is a travelling-wave structure, not a resonant one.

Exercise 6.6: The Corner Reflector

Purpose: *To illustrate the properties of the Corner Reflector, specifically the effect of the reflector.*

[Kraus, 1988, Pg549] first designed a corner reflector. They can come in many shapes and sizes, but the most common form is where the "corner" is defined as being 90°, and the reflector panels are simply made from vertical rods. At some distance from the corner, symmetrically placed is a dipole driven element.

1. Add a Corner Reflector by `Add| Assembly| antennas| sncorner` and change the feed spacing to 0.3m (`Feed distance from apex`). Add an xy plane radiation pattern under `Edit| Simulation Settings`. Simulate and plot, without closing the pattern viewer.
2. In the Input Viewer, click `Select All` and `Edit` to pull up the `sncorner` dialogue box again, and repeat the plot for a 0.7 by 0.7 reflector plate (`Side length` and `Reflector height` entries), overlaying it.
3. Repeat for a 2m by 2m reflector. You should get something like fig 6.20

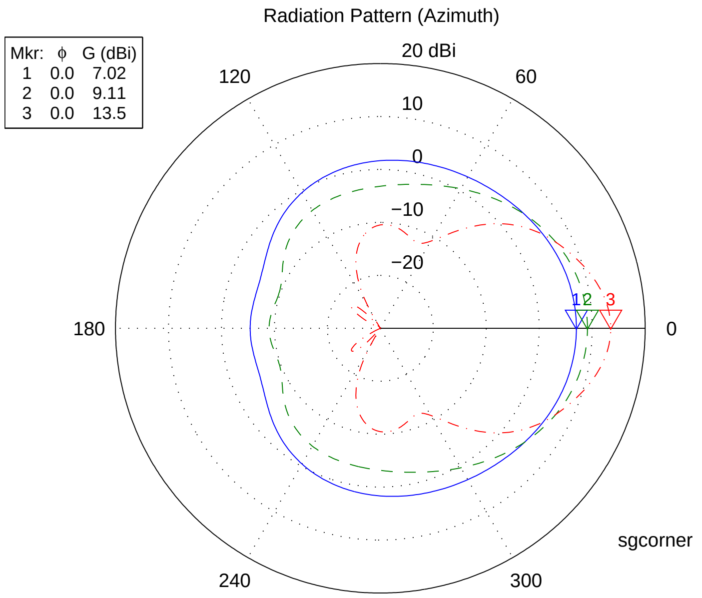

Figure 6.20: Corner reflector gain for increasing reflector size (1 = 0.5m; 2 = 0.7m; 3 = 2m)

Record the HPBW (Half Power Beam Widths obtained from the models in table 6.11. You will probably choose `View| Rectangular` in the pattern viewer, and then re-orientate the axes to position the peaks in the middle of the graph by `Options| Limits| Angle` and change the `[0 360]` to `[-180 180]`.

Table 6.11: Half Power Beam Widths of Corner Reflectors with varying sized panels

Panel Size	HPBW (degrees)
0.5m	
0.7m	
2m	

4. As an exercise in Mouse Madness, in the Input Viewer, set the `Model Frequency` to 400 MHz, remembering to click `Set`, use the < button to get the `Group Level` to `low`, and select all horizontal parts of the reflector screen with `Shift-Mouse 1` and hit `Delete`, leaving you with fig 6.21

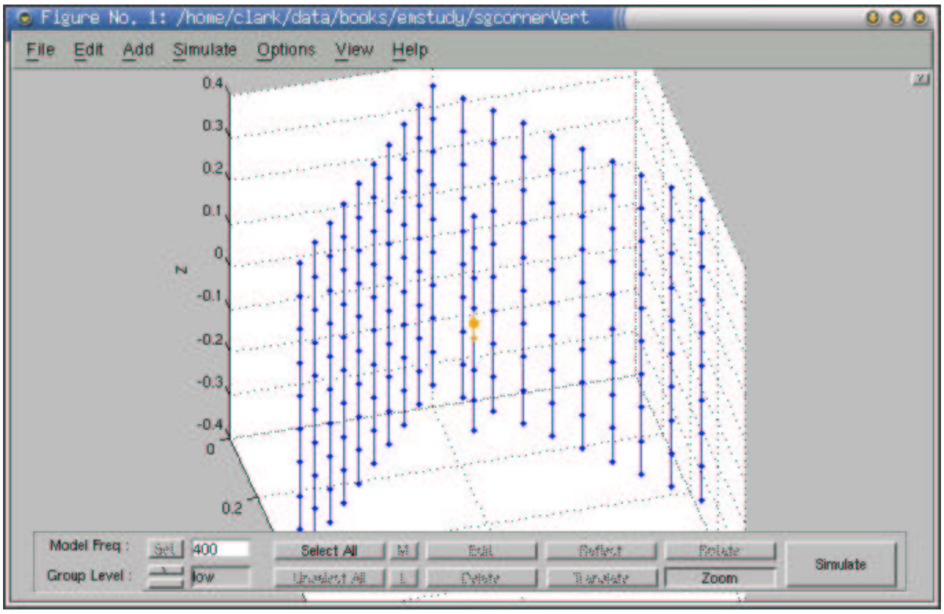

Figure 6.21: Corner Reflector with only vertical screen

The Difference in output is shown in fig 6.22.

5. Using the thinned down version (Save it! if you re-segment, by changing the model frequency, all those lovely horizontal segments come back!) `Edit| Simulation Settings` and set up a frequency Sweep from 200 to 400 MHz, edit the existing Radiation Pattern, pulling it down to only one point in the maximum gain direction. ie change `Frequency` to `[200:400]` and `Phi` from `[0,360,361]` to `[0,360,1]` Click `Modify`.

   Before you click the `Simulate` button, ensure that you exit the Output

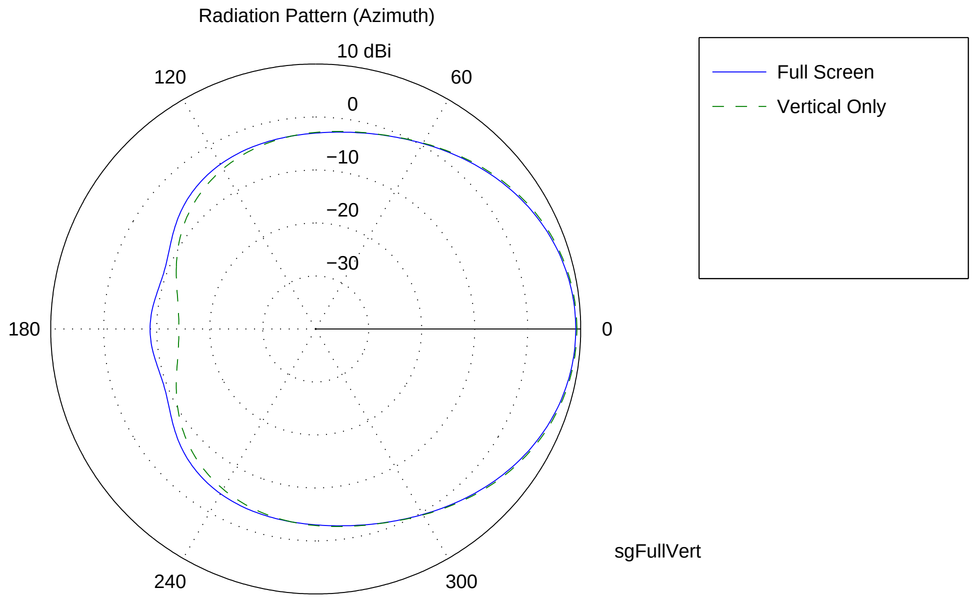

Figure 6.22: Difference between a full screen and a vertical only screen in the corner reflector

Viewer, as it gets confused when models with single frequency points and models with frequency sweeps are combined: it won't show the gain versus frequency.

Plot the gain versus frequency and the VSWR versus frequency, obtaining fig 6.23 and 6.24

Conclusion: It can be seen that the reflector size impacts on the forward gain of the Corner Reflector antenna, but that the most dramatic effect is on the backlobe. If you need a really good Front-to-Back Ratio, with no backward radiation, the corner reflector with a large screen is really good, but really expensive!

A decent compromise is shown as the "Clark Standard" design which uses a 0.7m reflector panel, which cuts the material cost. A further reduction in cost is to have vertical only screens, as shown in fig 6.22. (Cutting the simulation size from 358 segments to 178 segments, shortening the time too!)

Again, it is seen that the increased gain is obtained be squeezing the energy into a smaller portion of space, lowering the HPBW obtained as the gain increases. As usual, since the Corner Reflector is a Resonant Structure, it is impedance-bandwidth limited.

6.5 Problems

6-1. Yagi-Uda array design Design a 12-element Yagi-Uda array to work

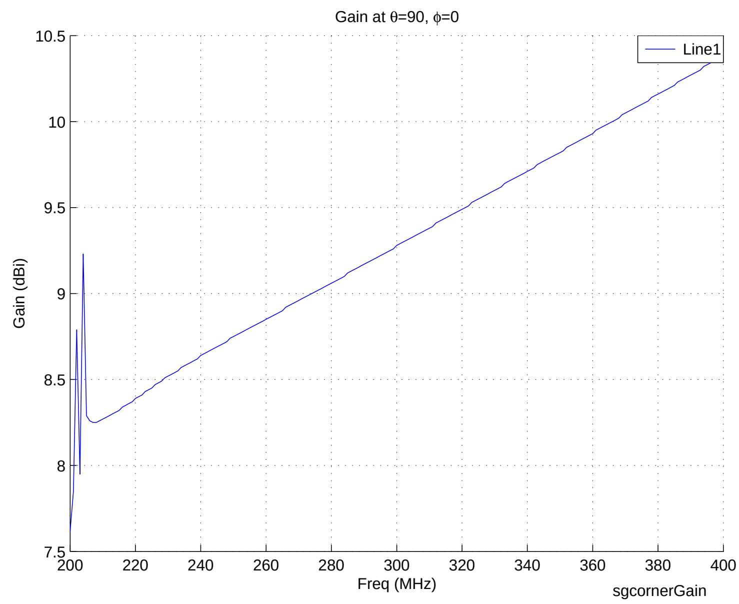

Figure 6.23: Corner Reflector Gain variation with frequency (ignore low end!)

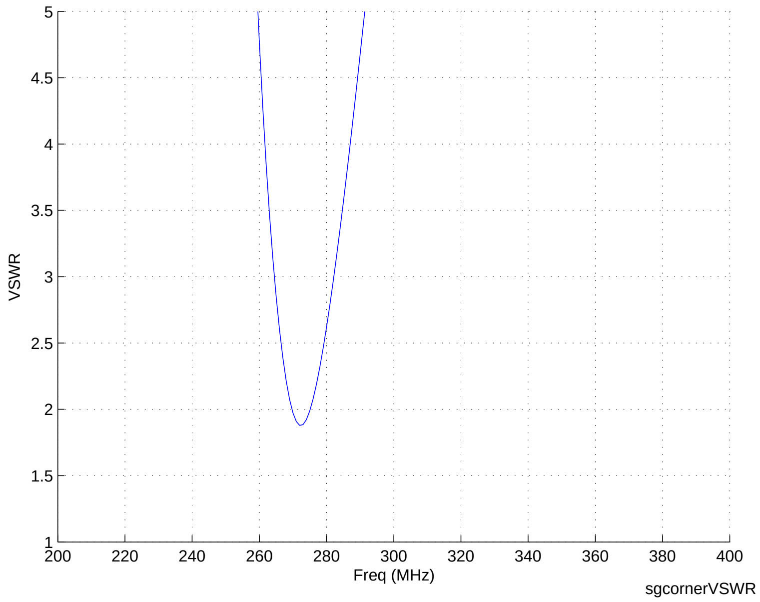

Figure 6.24: Corner Reflector VSWR variation with frequency

at the UHF band of frequencies (In Johannesburg, TV1,2,3 is at x,y,z MHz) Optimise the gain and VSWR bandwidths for these bands. Remember that Commercial television inputs have a characteristic impedance of 75Ω

6-2. Yagi-Uda array Matching Using any matching technique, improve the

VSWR 2:1 impedance bandwidth of the Yagi-Uda array of exercise 6.2.2. Compare your results to the commonly held view that "the 2:1 impedance bandwidth corresponds to the 1.5dB gain bandwidth".

6-3. LPDA Mess Investigate the gain "glitches" in an LPDA: Exit the Output Viewer to clear its ideas of frequency, `Edit| Simulation Settings` and do something like `[130:0.1:140]` as a frequency sweep right through the glitch.

6-4. LPDA Verify the LPDA design parameters shown in table 6.8.

6-5. Helix Construct a 10 turn, and 20 turn helix and compare their predicted Half-Power Beamwidths (HPBW) to the theory. Pull up the output file (the name.out file) and inspect the section entitled `- - - RADIATION PATTERNS - - -` and verify that the Boresight gain is Circularly Polarised.

6-6. Corner Reflector Matching The impedance bandwidth of the Corner Reflector shown in fig 6.24 is absolutely pathetic. Use any of the matching techniques in the Matching Chapter to improve this.

6-7. Exploring other Antenna types SuperNEC has a number of antenna assemblies not explored in this study guide. The general idea of looking at beamwidths, versus length of antenna, and gain & impedance bandwidth versus frequency is a good starting point for investigating an antenna's properties. Try this for a number of antenna assemblies not yet investigated.

6-8. Creating Assemblies Write a Corner Reflector assembly, based on `sncorner.m`, but without using `SIG` (Structure Interpolation and Gridding) in order to obtain a vertical-only screen.

6-9. Creating Assemblies Commercial Yagi-Uda arrays for domestic Television Reception often have a a vertical array of horizontal reflectors, ie instead of just having one reflector element as shown in the `snyagi` assembly, it might have three of them, or 5 of them in a vertical stack. Create an assembly which can create a user-defined number of such reflectors.

6-10. Creating Assemblies The idea behind specifying τ, σ pairs to design an LPDA is elegant from an academic perspective, but not very practical: Generally speaking, one of the main constraints that a designer wants to specify is the maximum boomlength, and it takes several iterations to get to the right ballpark. Create an assembly that takes the boomlength as a primary design input.

6-11. Using Created Assemblies Investigate the usefulness of the vertically stacked reflectors of 6.5 in

1. Enhancing forward gain,
2. Improving the front-to-back ratio. (Forward gain versus backward gain)

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Numbers in *italics* refer to figures; in **bold** to "main" references.

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